Article

Price adjustments by imperfectly competitive firms

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Abstract

This study examines the sufficient conditions guaranteeing the asymptotic stability of the general equilibrium for an economy under imperfect competition, in which firms adopt price and quantity strategies. The aim is to obtain a preliminary result to generalize the theorems in Sakane (2016).

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1. Introduction

This study examines the dynamics of prices for the general equilibrium under imperfect competition. Sakane (2016) proved that the general equilibrium for an economy with delays in a production process under perfect and imperfect competition in which firms adopt price strategies is asymptotically stable. Further, by comparing the stability conditions of the two models, it has also been verified that it is favorable that the slope of every demand curve is gradual (steep) for the asymptotic stability of the equilibrium under perfect (monopolistic) competition.

Nevertheless, two main subjects remain to be studied in this research area. First, the asymptotic stability of a general equilibrium for an economy with delays in a production process under imperfect competition when firms adopt quantity strategies must be proven. Second, it is also important to investigate the mutual relation between sufficient conditions guaranteeing the asymptotic stability of the equilibrium for an economy under

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(i) perfect competition; (ii) imperfect competition when price strategies are adopted; and (iii) imperfect competition when quantity strategies are adopted.

Without a time lag, we can obtain a preliminary result for the relation between the sufficient conditions for the asymptotic stability of the equilibrium under imperfect competition when price and quantity strategies are adopted.

2. Definitions

2.1. *Adjustment process adopted by imperfectly competitive firms using price policies*

We assume that there exist $\ell + 1$ types of commodities. For any $i \in \{1, \cdots, \ell+1\}$, q_i denotes the price of the ith commodity, which is a strictly positive real number, and q represents a vector $(q_1, \cdots, q_{\ell+1})$. Similarly, the prevailing price for the ith commodity at time t is denoted by $q_i(t)$ and its vector $(q_1(t), \cdots, q_{\ell+1}(t))$ is written as $q(t)$. Further, for any $i \in \{1, \cdots, \ell+1\}$, let $x_i : R^{\ell+1}_{++} \to R$ be the demand function for the ith commodity. The following assumption is set up about the function:

(A. 1) For any $i \in \{1, \cdots, \ell+1\}$:
(i) x_i is linear, that is,

$$x_i(q) := \sum_{j \in \{1, \cdots, \ell+1\}} a_{ij} q_j + b_i$$

for some coefficient a_{ij} for any $i \in \{1, \cdots, \ell+1\}$ and some constant $b_i > 0$;
(ii) $x_i(q)$ is homogeneous of degree zero for q; and
(iii) $a_{ij} > 0$ For any $i \in \{1, \cdots, \ell+1\}$ and $j \neq i$.

Under condition (ii), any price may be normalized as $p_j = q_j q_{\ell+1}^{-1}$ if the $\ell+1$th commodity is adopted by the numéraire. Moreover, a price vector may be defined by

$$p := (p_1, \cdots, p_\ell).$$

Thus, the quantity demanded for the ith commodity is hereafter denoted as $x_i(p)$ and its vector written as $x(p)$:

$$x_i(p) := \sum_{j \in \{1, \cdots, \ell\}} a_{ij} p_j + a_{i\ell+1} + b_i. \tag{1}$$

Condition (A. 1)(iii) means that all commodities are substitutes. According to Euler's theorem, $a_{ii} < 0$ under (A. 1)(ii) and (iii).

We suppose that there exist ℓ monopolistically competitive firms that produce one differentiated product by inputting one homogeneous production factor ($\ell + 1$th commodity) provided by consumers and that only supply their products to consumers. For any firm $i \in \{1, \cdots, \ell\}$, the technology is assumed to be represented by the production function $f_i : R_+ \to R$. We set up the following assumption about the function:

(A. 2) For any $j \in \{1, \cdots, \ell\}$:
(i) $f_j : R_+ \to R$ is twice differentiable for any $\omega \in R_{++}$;
(ii) $\lim_{\omega \to 0} f_j(\omega) = 0$; and
(iii) $f_j'(\omega) > 0$ and $f_j''(\omega) < 0$ for any $\omega \in R_{++}$.

We also assume that the demand function depends only on prices and that any profit that each firm distributes to consumers has an insignificant effect on the quantity demanded. The reason why the profit effect must be omitted is described in Sakane (2016). Since the wealth of consumers is a constant variable for each firm under this assumption, the quantity demanded depends only on prices. Given the assumptions above, the profit function $m_i : R^{\ell}_{++} \to R$ for the ith firm is defined as follows:

$$m_i(p) := x_i(p)p_i - f_i^{-1}(x_i(p)).$$

The value $m_i(p)$ is not always the maximum since the process through which each firm gradually improves its profitability is considered in this model. Since we assume that every firm can satisfy the condition $y_i = x_i(p)$ for any $p \in P$,

$$\frac{\partial f_i^{-1}(x_i(p))}{\partial p_i} = f_i^{-1\prime}(y_i)\frac{\partial x_i(p)}{\partial p_i}.$$

Since $\partial m_i(p^*)/\partial p_i = a_{ii}p_i^* + \sum_{j\in\{1,\cdots,\ell\}} a_{ij}p_j^* + a_{i\ell+1} + b_i - f_i^{-1\prime}(y_i)a_{ii} = 0$ and $f_i^{-1\prime\prime}(y_i)$ is constant under (A. 2)(i), which is denoted by c_i, it follows that

$$\frac{\partial m_i(p)}{\partial p_i} = a_{ii}(p_i - p_i^*) + \sum_{j\in\{1,\cdots,\ell\}} a_{ij}(1 - a_{ii}c_i)(p_j - p_j^*)$$

for any $i \in \{1, \cdots, \ell\}$. Thus, the price adjustment process is defined as follows:

$$\frac{dp_i}{dt} = a_{ii}(p_i - p_i^*) + \sum_{j\in\{1,\cdots,\ell\}} a_{ij}(1 - a_{ii}c_i)(p_j - p_j^*) \tag{2}$$

for any $i \in \{1, \cdots, \ell\}$.

Suppose that for any $i \in \{1, \cdots, \ell\}$, α_{ij} is defined by

$$\alpha_{ij} := \begin{cases} a_{ij} + a_{ij}(1 - a_{ii}c_i) & j = i \\ a_{ij}(1 - a_{ii}c_i) & j \neq i \end{cases} \tag{3}$$

and A_α is an (ℓ, ℓ)-matrix defined as follows:

$$A_\alpha := \big(\alpha_{ij}\big).$$

The previous differential equation (2) is denoted as follows:

$$\frac{dp(t)}{dt} = A_\alpha(p - p^*). \tag{4}$$

2.2. *Adjustment process adopted by monopolistically competitive firms using quantity policies*

Suppose that A is an (ℓ, ℓ)-matrix defined by

$$A := \big(a_{ij}\big),$$

in which a_{ij} is a coefficient in the definition (1) of the demand function and that $x(p)$, $a_{\ell+1}$ and b are row vectors $(x_1(p), \cdots, x_\ell(p))$, $(a_{1\ell+1}, \cdots, a_{\ell\ell+1})$, and $(b_1, \cdots, b_\ell)$, respectively. Then, the matrix representation of the demand function (1) is as follows:

$$x(p) := Ap + a_{\ell+1} + b.$$

For any list $y := (y_1, \cdots, y_\ell)$ of production plans for firms, we assume that a price vector p satisfying the condition $Ap + a_{\ell+1} + b = y$ is uniquely determined, that is,

$$p = A^{-1}(y - a_{\ell+1} - b).$$

Let $\tilde{a}_{ij}$ be an (i, j)-cofactor of matrix A. The ith element p_i of (2.2), which is denoted as $p_i(y)$ as a function of y, is as follows:

$$p_i(y) = \frac{1}{|A|} \sum_{j \in \{1, \cdots, \ell\}} \tilde{a}_{ji}(y_j - a_{j\ell+1} - b_j).$$

As a result, we can define the profit of the ith monopolistically competitive firm by using quantity strategies as follows:

$$\hat{m}_i(y) := p_i(y)y_i - f_i^{-1}(y_i).$$

Since $\partial \hat{m}_i(y^*)/\partial y_i = |A|^{-1}\tilde{a}_{ii}y_i^* + |A|^{-1}\sum_{j \in \{1, \cdots, \ell\}} \tilde{a}_{ji}(y_j^* - a_{j\ell+1} - b_j) - {f_i^{-1}}'(y_i^*) = 0$,

$$\frac{\partial \hat{m}_i(y)}{\partial y_i} = \left(\frac{\tilde{a}_{ii}}{|A|} - c_i\right)(y_i - y_i^*) + \frac{1}{|A|} \sum_{j \in \{1, \cdots, \ell\}} \tilde{a}_{ji}(y_j - y_j^*)$$

for any $i \in \{1, \cdots, \ell\}$. Thus, we consider the following quantity-adjustment process:

$$\frac{dy_i(t)}{dt} = \left(\frac{\tilde{a}_{ii}}{|A|} - c_i\right)(y_i - y_i^*) + \frac{1}{|A|} \sum_{j \in \{1, \cdots, \ell\}} \tilde{a}_{ji}(y_j - y_j^*) \tag{5}$$

for any $i \in \{1, \cdots, \ell\}$.

Suppose that for any $i \in \{1, \cdots, \ell\}$, β_{ij} is defined by

$$\beta_{ij} := \begin{cases} \dfrac{\tilde{a}_{ji}}{|A|} + \dfrac{\tilde{a}_{ii}}{|A|} - c_i & j = i \\ \dfrac{\tilde{a}_{ji}}{|A|} & j \neq i \end{cases} \tag{6}$$

and A_β is an (ℓ, ℓ)-matrix defined as follows:

$$A_\beta := \big(\beta_{ij}\big).$$

Then, the previous differential equation (5) is denoted as follows:

$$\frac{dy(t)}{dt} = A_\beta(y - y^*). \tag{7}$$

3. Relation between the two models

We now consider the initial-value problem with respect to the previous equation (4) by imposing the initial condition $p(0) = p^o$:

$$\frac{dp(t)}{dt} = A_\alpha(p - p^*), \quad p(0) = p^o. \tag{8}$$

The problem can be solved as follows:

$$p(t) = \exp(tA_\alpha)(p^o - p^*) + p^*. \tag{9}$$

In the same way, the solution to the initial-value problem

$$\frac{dy(t)}{dt} = A_\beta(y - y^*), \quad y(0) = y^o \tag{10}$$

with respect to (7) is as follows:

$$y(t) = \exp(tA_\beta)(y^o - y^*) + y^*. \tag{11}$$

We therefore establish the following theorem on the relation between the solutions (9) and (11).

Theorem. *Suppose that $p(t)$ and $y(t)$ are solutions and p^* and y^* are equilibria of the initial-value problrms (8) and (10), respectively and that for any $i \in \{1, \cdots, \ell\}$ and $j \in \{1, \cdots, \ell\}$ $(i \neq j)$, $a_{ij} = 0$. Then, for every $i \in \{1, \cdots, \ell\}$,*
(i) *if $|a_{ii}| < 1$, $p(t)$ converges to p^* faster than that $y(t)$ converges to y^*;*
(ii) *if $|a_{ii}| > 1$, $y(t)$ converges to y^* faster than that $p(t)$ converges to p^*; and*
(iii) *if $|a_{ii}| = 1$, velocities of convergence for $y(t)$ to y^* and $p(t)$ to p^* are identical.*

Proof. Suppose that C and D are diagonal matrices defined as follows, respectively:

$$C := \begin{pmatrix} 2 - a_{11}c_1 & & 0 \\ & \ddots & \\ 0 & & 2 - a_{\ell\ell}c_\ell \end{pmatrix}$$

and that

$$D := \begin{pmatrix} a_{11} & & 0 \\ & \ddots & \\ 0 & & a_{\ell\ell} \end{pmatrix}.$$

Then, A_α may be redefined as follows:

$$A_\alpha := CD$$

By the same way, matrix A_β is also redefined as follows:

$$A_\beta = CD^{-1}.$$

Now, we may turn to a comparison between the solutions (9) and (11). It follows that

$$\frac{\|p(t) - p^*\|}{\|y(t) - y^*\|} = \frac{\|\exp(tA_\alpha)(p^o - p^*)\|}{\|\exp(tA_\beta)(y^o - y^*)\|} = \frac{\|\exp(tCD)(p^o - p^*)\|}{\|\exp(tCD^{-1})(y^o - y^*)\|}.$$

As $\exp(tCD)$ is defined by $tI + tCD + (2\,!)^{-1}\,(tCD)^2 + \cdots$, the (i,i)-component of the matrix is as follows:

$$1 + t(2 - a_{ii}c_i)a_{ii} + \frac{t}{2\,!}(2 - a_{ii}c_i)^2 a_{ii}^2 + \cdots.$$

And it is also clear that the (i,j)-component of the matrix is 0 if $i \neq j$.

By the same way, since $\exp(tA_\beta)$ is defined by $(tCD^{-1}) + (2\,!)^{-1}\,(tCD^{-1})^2 + \cdots$, the (i,i)-component of the matrix is as follows:

$$1 + t(2 - a_{ii}c_i)\frac{1}{a_{ii}} + \frac{t^2}{2\,!}(2 - a_{ii}c_i)^2\left(\frac{1}{a_{ii}}\right)^2 + \cdots .$$

The (i,j)-element of the matrix is clearly 0 if $i \neq j$. Therefore, if $|a_{ii}| < 1$ for any $i \in \{1, \cdots, \ell\}$, then as $t \to \infty$,

$$\frac{\|\exp(tCD)(p^o - p^*)\|}{\|\exp(tCD^{-1})(y^o - y^*)\|} \to 0.$$

This verifies condition (i). Conditions (ii) and (iii) are also clear. □

References

Sakane, H., 2016. Asymptotic stability of a general equilibrium under perfect and monopolistic competition. Journal of Mathematical Economics 63, 21–26.