*Kansai University Review of Economics*
No.19 (March 2017), pp.1–7

# Business cycle accounting for the Japanese economy using the parameterized expectations algorithm

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December 1, 2016

**Abstract**

We propose an application of the parameterized expectations algorithm (PEA) to business cycle accounting (BCA). The PEA has an advantage in that it is simple and easier to understand and implement than other non-linear solution methods for a dynamic stochastic general equilibrium model. Moreover, we apply BCA to the Japanese economy using the PEA, which relaxes the perfect foresight assumption and yields a result similar to the main finding of deterministic BCA by Kobayashi and Inaba (2006). The effects of the investment wedge are not a significant cause of the persistent recession during the 1990s. The output derived from the efficiency wedge roughly replicates actual output, while the discrepancy widened during the 1990s. The labor wedge had a significant depressing effect on output during 1989-2005. The efficiency wedge explains the recent economic recovery.

**Keywords:** Business cycle accounting, parameterized expectation algorithm

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# Introduction

Chari, Kehoe, and McGrattan (2002, 2004, 2007a) proposed business cycle accounting (BCA) to assess which wedge is important in the fluctuations of an economy under the assumption that it is a prototype model with time-varying wedges. These wedges resemble productivity, labor, and investment taxes, and government consumption. Since researchers measure these wedges using the production function and first order conditions to fit the actual macroeconomic data, this method can represent a generalization of the growth accounting.

In this study, we apply the parameterized expectations algorithm (PEA) to BCA. This study has two contributions. The first is an application of the PEA to BCA. The PEA proposed by Marcet (1988) is one method to solve the non-linear dynamic stochastic general equilibrium model. Marcet and Lorenzoni (1998) provide an application of the PEA to some economic models. The PEA essentially approximates the expectation function with a smooth function, a polynomial function. The PEA has an advantage[1] in that it is simpler and easier to understand and implement than other non-linear solution methods.[2]

Second, we apply BCA to the Japanese economy using the PEA, which relaxes the perfect foresight assumption and yields a result similar to main result for a deterministic BCA by Kobayashi and Inaba (2006). They assume perfect foresight in the prototype economy so that all wedges are given deterministically, as in Chari et al. (2002). We can use the perfect foresight assumption to avoid complicated calculations. As they point out, however, the effects of the investment wedge are sensitive to the assumption of the future values of wedges.[3] On the other hand, the stochastic model that assumes that the wedges are an exogenous stochastic process estimated from the data does not suffer from the arbitrary choices of the future values of wedges. Chakraborty (2004) also applies BCA to the Japanese economy using a log-linearized dynamic stochastic general equilibrium model. The simulation result for the investment wedge is somewhat different from Kobayashi and Inaba (2006). In this study, we find that the BCA result using PEA is similar to that from the perfect foresight BCA. Therefore, we can conclude that the difference in the results between Chakraborty and Kobayashi-Inaba must stem from the data constructions, data sources, and log-linearization. In cases where the economy is far from a steady state or highly non-linear, the approximation error may be large. Therefore, accounting for non-linearities may be important.

The organization of the paper is as follows. Section 2 describes the prototype model in BCA. Section 3 explains the accounting procedure using the PEA. Section 4 describes application of BCA with the PEA to the Japanese economy to investigate the robustness of results in Kobayashi and Inaba (2006). Section 5 concludes. Similar to the main results of Kobayashi and Inaba (2006), this study finds that the effects of the investment wedge were not a significant cause of the persistent recession during the 1990s. The output due to the efficiency wedge roughly replicates actual output, while the discrepancy widened during the 1990s. The labor wedge had a significant depressing effect on output during 1989-2005. The efficiency wedge explains the recent economic recovery.

# The prototype model

This section describes the prototype model with time-varying wedges: the efficiency wedge $A_t$, the labor wedge $1-\tau_{l,t}$, the investment wedge $1/(1+\tau_{x,t})$, and the government wedge $g_t$.

The household maximizes

$$\max_{c_t, k_{t+1}, l_t} E_0 \left[ \sum_{t=0}^{\infty} \beta^t U(c_t, l_t) N_t \right]$$

[1]The PEA also has a disadvantage in that it requires a long simulation to obtain the fitted coefficients of the approximating function. Thus, the algorithm can be quite computationally demanding.

[2]Chari et al. (2004, 2007) implement BCA using the finite element method for the non-linear solution described by McGrattan (1996).

[3]In the perfect foresight assumption, the future value of wedges is arbitrary. Kobayashi and Inaba (2006) check the following four cases: (1) all wedges remain constant at the last value of the target period; (2) the labor and investment wedges are zero in the future and the efficiency wedge is the benchmark value, i.e., the 1984-1989 average; (3) the labor and investment wedges are zero in the future and the efficiency wedge is the last value of the target period; and (4) all wedges are the benchmark values in the future.

subject to

$$c_t + (1+\tau_{x,t})\left\{\frac{N_{t+1}}{N_t}k_{t+1} - k_t\right\} = (1-\tau_{l,t})w_t l_t + r_t k_t + T_t, \quad 0 < \beta < 1,$$

where $c_t$ denotes consumption, $l_t$ employment, $N_t$ population, $k_t$ capital stock, $w_t$ the wage rate, $r_t$ the rental rate on capital, and $T_t$ the lump-sum taxes per capita. All quantities in lower-case letters denote per-capita quantities, except for $T_t$.

The firm maximizes

$$\max_{k_t,l_t} A_t F(k_t, \gamma^t l_t) - \{r_t + (1+\tau_{x,t})\delta\}k_t - w_t l_t,$$

where $\delta$ denotes the depreciation of capital stock and $\gamma$ the balanced growth rate of technological progress. The resource constraint is

$$c_t + x_t + g_t = y_t, \tag{1}$$

where $x_t$ is investment, $g_t$ the government consumption, and $y_t$ the per-capita output. The law of motion for capital stock is

$$\frac{N_{t+1}}{N_t}k_{t+1} = (1-\delta)k_t + x_t. \tag{2}$$

The equilibrium is summarized by the resource constraint (1), the law of motion capital (2), the production function,

$$y_t = A_t F(k_t, \gamma^t l_t), \tag{3}$$

and the first-order conditions,

$$-\frac{U_{l,t}}{U_{c,t}} = (1-\tau_{l,t})A_t\gamma^t F_{l,t}, \tag{4}$$

$$U_{c,t}(1+\tau_{x,t}) = \beta E_t U_{c,t+1}\left[A_{t+1}F_{k,t+1} + (1-\delta)(1+\tau_{x,t+1})\right], \tag{5}$$

where $U_{ct}$, $U_{lt}$, $F_{lt}$, and $F_{kt}$ denote the derivatives of the utility function and the production function with respect to their arguments. The functional form of the utility function is $U(c,l) = \ln c + \phi\ln(1-l)$, where $\phi > 0$ is a parameter. The functional form of the production function is $F(k,l) = k^{\alpha}l^{1-\alpha}$.

# Accounting procedure

This section describes the accounting procedure to measure the actual wedges using PEA.

## Measuring the wedges

We take the government wedge $\tau_g$ directly from the data. For the other wedges, we obtain the values using data for $y_t$, $l_t$, $x_t$, $g_t$, and $N_t$, together with a series for $k_t$ constructed from $x_t$ by (2). We calculate the efficiency and labor wedges directly from (3) and (4).

To find the investment wedge $\tau_{x,t}$, Kobayashi and Inaba (2006) assume a deterministic model and posit a strict assumption on the values of the wedges for the period after the target BCA period.[4] As they point out, however, the effects of the investment wedge depend on the assumption for the values of future wedges.

In this study, we apply the PEA to find the investment wedge $\tau_{x,t}$. The algorithm is as follows.

### Algorithm [5]

- **Initialization**: *Apply the deterministic method of BCA (Kobayashi and Inaba, 2006), take the derived investment wedge as the initial value of* $\tau_{x,t}^{(0)}$*, and set a stopping parameter* $\epsilon > 0$

[4] Their procedure is as follows. Denoting the target BCA period by $t = 0,1,2,...,T$, they assume that $A_t = A^* = A_T$, $\tau_{l,t} = \tau_l^* = \tau_{l,T}$, and $g_t = g^* = g_T$ for $t \geq T+1$. They also assume that $\tau_{x,t}$ is an unknown constant $\tau_x^*$ for $t \geq T$, and use the shooting method to find $\tau_x^*$ such that $\tau_{x,T} = \tau_{x,T+1} = \tau_x^*$. After determining $\tau_x^* = \tau_{x,T}$ by this method, the authors obtain $\tau_{x,t}$ for $t = 0,1,2,...,T-1$ by solving (5) backward.

[5] For details, see the technical appendices (Inaba, 2007).

- **Step 1**: *Specify a vector AR1 process for the four wedges* $s_t = (\log(A_t), \tau_{l,t}, \tau_{x,t}^{(j)}, \log(g_t))'$ *of the form*

$$s_{t+1} = P_0 + P s_t + \eta_{t+1}, \tag{6}$$

*where* $\eta_t \sim$ *i.i.d.* $N(0, \Omega)$.[6]

- **Step 2**: *Apply the parameterized expectation algorithm to obtain the non-linear solution to the model. We then get an approximation function* $\Phi(\cdot)$ *for the expectation function*[7]*:*

$$\beta E_t U_{c,t+1} \left\{ A_{t+1} F_{k,t+1} + (1-\delta)(1+\tau_{x,t+1}^{(j)}) \right\}.$$

$\Phi(\cdot)$ *is a polynomial function of* $k_t$, $A_t$, $\tau_{l,t}$, $\tau_{x,t}^{(j)}$, *and* $g_t$.

- **Step 3**: *To find the value of* $\hat{\tau}_{x,t}$ *that reflects the actual data,* $c_t$ *and* $l_t$*, solve the following equation for* $\hat{\tau}_{x,t}$*:*

$$U_{c,t}(1+\hat{\tau}_{x,t}) = \Phi(k_t, A_t, \tau_{l,t}, \hat{\tau}_{x,t}, g_t) \tag{7}$$

- **Step 4**: $\tau_{x,t}^{(j+1)} = \nu \hat{\tau}_{x,t} + (1-\nu)\tau_{x,t}^{(j)}, \quad 0 < \nu < 1.$

- **Step 5**: *If* $\| \tau_{x,t}^{(j+1)} - \tau_{x,t}^{(j)} \| < \epsilon$, *STOP; else, go to step 1.*

## Decomposition

In order to see the effect of the measured wedges on movements in macroeconomic variables from an initial date $t = 0$, we decompose the movements as in Chari, Kehoe, and McGrattan (2007a)[8]. For example, to evaluate the effects of the efficiency wedge, we compute the decision rules for an economy with only the efficiency wedge, denoted with $y^e(s_t, k_t)$, $c^e(s_t, k_t)$, $l^e(s_t, k_t)$, and $x^e(s_t, k_t)$ under an exogenous stochastic process that we assume to be the combination of (6) and a one-to-one mapping function:

$$\log A(s_t) = \log A_t,\ \tau_l(s_t) = \bar{\tau}_l,\ \tau_x(s_t) = \bar{\tau}_x,\ \log g(s_t) = \log \bar{g}. \tag{8}$$

Starting from initial value of capital stock, $k_0$, we then use the actual value of wedges, $s_t$, the decision rules for the economy with only the efficiency wedge, and the capital accumulation law to compute the realized sequence of output, consumption, labor, and investment, $y_t^e$, $c_t^e$, $l_t^e$, and $x_t^e$, respectively, which we call the *efficiency wedge components* of output, consumption, labor, and investment. Similarly, we define the *labor wedge components*, *investment wedge components*, *government wedge components*, and *benchmark components*.

We compare the effect of each wedge as follows. First, we construct the benchmark components by solving the prototype model with constant wedges. We choose the values of the benchmark wedges as the initial values at $t = 0$, or the averages of the values of the wedges for some period prior to the target period. Therefore, we solve the model assuming that $s_t$ is a constant vector consisting of the benchmark wedges. We take the derived sequences $y_t^b$, $c_t^b$, $x_t^b$, and $l_t^b$ as the benchmark case. We then compare each component of output with the benchmark components. If the derived output is below the benchmark, we say that the efficiency wedge has a depressing effect compared to the benchmark case.

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[6] The OLS estimation of this stochastic process can be non-stationary. Therefore, we use maximum likelihood procedure described in McGrattan (1994) to estimate the parameters $P_0$ and $P$ of the vector AR1 process for the wedges. To ensure stationarity, we add a penalty term proportional to $\{\max(|\lambda_{\max}| - 0.99, 0)\}^2$ to the likelihood function, where $\lambda_{\max}$ is the maximal eigenvalue of $P$. If $\lambda_{\max} < 0.99$, we use the OLS estimation.

[7] The main drawback of the PEA is that it is not a contraction mapping technique and does not guarantee a solution. We avoid this by modifying the PEA following Maliar and Maliar (2003). They discuss a moving bounds method of imposing stability on the PEA to avoid the explosive case due to poor initial parameter values and to enhance the PCA's convergence property.

[8] We also implement the alternative decomposition in Chari, Kehoe, and McGrattan (2004) and find similar results. For details, see the technical appendices in Inaba (2007). Chari, Kehoe, and McGrattan (2007b) explain the differences between the two decompositions.

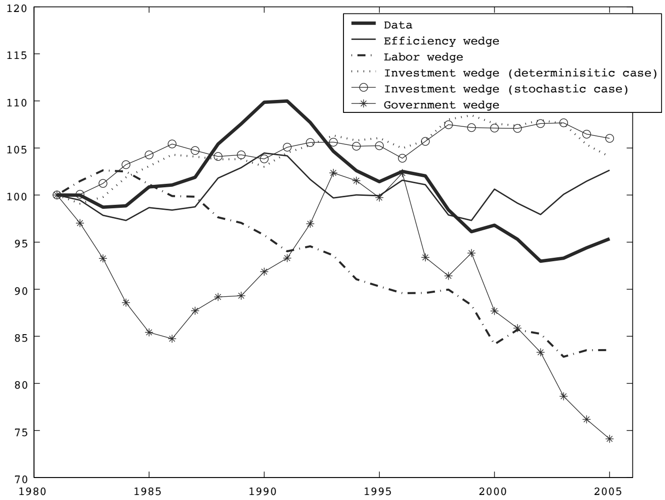


Figure 1: Output and the four wedges (100 in 1981)

## BCA for Japan

The target period of our accounting exercise is 1981-2005. We update[9] the data set constructed by Kobayashi and Inaba (2005) and use the same assumptions as Kobayashi and Inaba (2006), except for the accounting algorithm. We set $\beta = 0.98$, $\alpha = 0.372$, and $\delta = 0.0892$, which are the averages during 1984-1989, except for $\beta$. We also set $g_n = 0$, and $g_z = 0.0214$, where $g_n$ is the population growth rate, and $(1 + g_z)^{1-\alpha} = \gamma$ for the simulation. The trend rate of technological progress $(1 + g_z)$ and we set this as the average during 1981-2005.

In figure 1 we illustrate the actual output data (detrended by $1 + g_z$) and the four measured wedges for 1981-2005: the efficiency wedge $A_t$, the labor wedge $(1 - \tau_{l,t})$, the investment wedge $1/(1 + \tau_{x,t})$, and the government wedge $g_t$. We plot all variables as indices set at 100 in 1981. The fluctuations in the investment wedge derived by the PEA are quite similar to those of the deterministic case.

Figure 2 shows the decomposition results for output. In our decomposition exercise, we assumed the following values of the benchmark wedges: $A$, $\tau_l$, $\tau_x$, and $g$ are the averages for the 1984-1989 period. In figure 2, we display the separate contributions of each wedge. We plot the actual output, benchmark case, and simulated outputs from each of the four wedges. We plot the benchmark as a horizontal line at 100 and the other outputs as deviations from the benchmark. If output due to a wedge is below (above) the benchmark case, we determine that the wedge has a depressing (uplifting) effect on output. The result is quite similar to that in Kobayashi and Inaba (2006). The effects of the investment wedge were not a significant cause of the persistent recession during the 1990s. The output due to the efficiency wedge roughly replicates actual output, while the discrepancy widened during the 1990s. The labor wedge had a significant depressing effect on output during 1989-2005. The efficiency wedge explains the recent economic recovery.

## Concluding remarks

This paper proposes an application of the parameterized expectation algorithm to business cycle accounting. The PEA is a simple algorithm and easier to understand and implement than the other non-linear solution methods. Moreover, under a less arbitrary assumption about the process of wedges than the perfect foresight BCA, we show that the BCA results using the PEA is similar to the main result in Kobayashi and Inaba (2006) deterministic BCA.

[9]While Kobayashi and Inaba (2006) use “private final consumption expenditure” as consumption, we use “actual final consumption of households”.

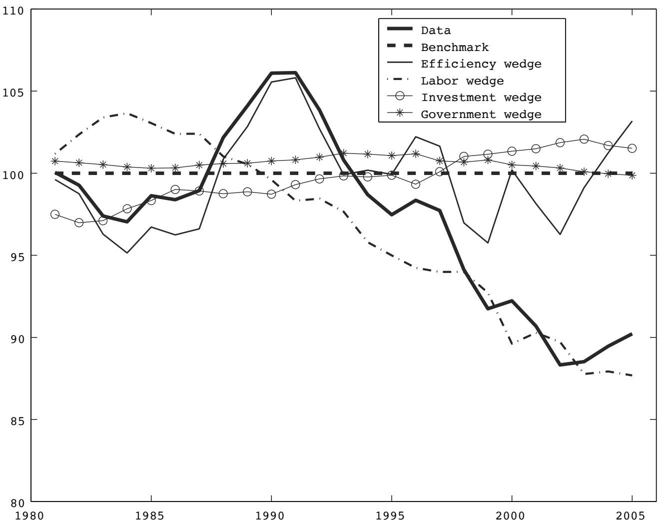


Figure 2: Decomposition of output with one wedge

## Acknowledgments

I would like to thank Keiichiro Kobayashi and Kengo Nutahara for their very helpful comments. All remaining errors are mine.

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