Kansai University Review of Economics
No. 11 (March 2009), pp. 55–71

Existence of equilibria for a monopolistically competitive economy

Hirokazu Sakane*

The aim of the paper is to establish the existence theorem on general equilibria for a large economy under monopolistic competition in which there exist uncountably infinite differentiated commodities.

Keywords: General equilibria, Monopolistic competition.

1 Introduction

In this paper, we study a large economy with infinitely many differentiated commodities under imperfect competition. Many papers discussing the existence of general equilibria hypothesize that numerous insignificant economic agents cannot influence markets – this is part of the assumption of perfect competition. Realistically, however, it is quite obvious that imperfectly competitive firms exist, and that they can manipulate prices. For this reason, we emphasize the importance of analyzing general equilibrium models with firms that can change prices. Negishi (1961) proved the existence of equilibria for an economy in which the market structure is monopolistically competitive. Since this pioneering paper, several papers studying general equilibrium models under imperfect competition have been presented. Early contributions were made by Arrow and Hahn (1971), Gabszewicz and Vial (1972), Fitzroy (1974), Marschak and Selten (1974), Laffont and Laroque (1976), Cornwall (1977), Silvestre (1977a), and Silvestre (1977b). Following these, studies by Hart (1979) and Hart (1985) investigated a large economy with differentiated commodities in relation to monopolistic competition.

* E-mail address: sakane@ipcku.kansai-u.ac.jp (Hirokazu Sakane) Tel.: +81-6-6368-0604.

Furthermore, as an application of a nonatomic game, Pascoa (1993) proved the existence of ε-equilibria for a monopolistically competitive economy. In addition to these monopolistically competitive models, an oligopolistically competitive model with a continuum of traders was presented by Codognato and Gabszewicz (1993), in which some of the agents behave strategically as price setters, while others remain price takers. Relevant to the concept of general equilibria with rigid prices, a monopolistically competitive model was studied by Benassy (1988). Although we do not make mention of this theme in our paper, it is significant in examining the relation of fixed price equilibria to the price making behavior of imperfectly competitive firms.

Distinctive features of our model are as follows. (i) As for models presented by Hart (1985) and Pascoa (1993), it is supposed that infinitely many commodities are traded. Thus, our model is more general than the basic models analyzed in the early literature. (ii) We define a large economy under monopolistic competition without a replicating method which is used in Hart (1985). (iii) As we consider a production economy, rather than the pure exchange economy studied in Codognato and Gabszewicz (1993), our model is more comprehensive. (iv) We do not study the existence of ε-equilibria, investigated by Hart (1985) and Pascoa (1993), but exact equilibria for our model.

The purpose of the paper is to establish the existence theorem on the general equilibria for a large economy under monopolistic competition. To this end, we must analyze price setting behavior by monopolistically competitive firms. When we do so, we confront some difficulties. In general, it is assumed in this research area that firms are able to manipulate prices along demand curves for their products. However, if the firms do not have sufficient productive capacity to satisfy the demands, then their production plans cannot be realized under prices that they charged. We can deal successfully with this problem by setting up a natural assumtion about the behavior of the firms.

2 De scription of a monopolistically competitive economy

2.1 Commodities and prices

We hypothesize that there exist infinitely many differentiated commodities or brands classified into ℓ categories and the numéraire commodity. For any $j \in \{1, \cdots, \ell\}$, let C_j be the set of all the differentiated commodities belonging to the j-th category. And, let c_j denote a typical element of the set. We assume that C_j is a nonempty, compact metric space, and that the set satisfies the following properties: (i) $C_1, \cdots, C_h$ are the sets of products firms can supply only to consumers – the firms cannot use the commodities $c_1, \cdots, c_h$ as production factors. (ii) $C_{h+1}, \cdots, C_\ell$ are the sets of initial endowments consumers can provide only for firms as production factors. Besides these commodities, there exists one homogeneous commodity. For convenience, we consider it as the ℓ+1-th commodity. In case of necessity, we denote it as $c_{\ell+1}$. We assume that $c_{\ell+1}$ is also an initial endowment of consumers, which is used only as a production factor. And we suppose that this commodity plays the role of numéraire. Let $\mathbb{C}_j$ be the collection of Borel subsets of C_j, and σ_j be a Borel measure defined on $\mathbb{C}_j$. The measure space of all differentiated commodities in the j-th category is specified by $(C_j, \mathbb{C}_j, \sigma_j)$. Further, we define $C := C_1 \times \cdots \times C_\ell$, $\mathbb{C} := \mathbb{C}_1 \otimes \cdots \otimes \mathbb{C}_\ell$ and $\sigma := \sigma_1 \times \cdots \times \sigma_\ell$.

Let q_j or $p_j\,(c_j)$ denote a relative price of commodity c_j and the ℓ+1-th numéraire commodity whose price is 1. We provisionally assume that the range of prices for c_j is a closed interval $[a_j, b_j]$, satisfying $0 < a_j < b_j < \infty$. We will drop the bounds of the interval in Section 3. Furthermore, we consider the product set $[a_1, b_1] \times \cdots \times [a_\ell, b_\ell]$, and denote a typical element of the set as $q := (q_1, \cdots, q_\ell)$ or $p(c) := (p_1(c_j), \cdots, p_\ell(c_j))$. Suppose that $L_1(C_j, R)$ is the whole set of all equivalence classes of measurable functions $p_j : C_j \to R$ with the norm $\|p_j\| := \int_{Cj} |p_j|\,\sigma_j$, then we define a nonempty subset $\mathfrak{P}_j$ of $L_1(C_j, R)$ as follows:

$$\mathfrak{P}_j := \{p_j \in L_1(C_j, R) | p_j(c_j) \in [a_j, b_j],\ a.e.\}. \tag{1}$$

It is shown that $\mathfrak{P}_j$ is compact for the norm $\| \cdot \|$. Let $\hat{\mathfrak{P}}_j$ be the set of all continuous functions $\hat{p}_j : C_j \to R$ satisfying $\hat{p}_j(c_j) \in [a_j, b_j]$ for any $c_j \in C_j$. It is clear that $\hat{\mathfrak{P}}_j$ is compact for the sup norm $\sup_{c_j \in C_j} |\hat{p}_j(c_j)|$. Thus, for any $\varepsilon \in (0,\infty)$, there exists $\bar{n} \in \{1, 2, \cdots\}$ such that $n \geqq \bar{n} \Rightarrow \sup_{c_j \in C_j} |\hat{p}_j^n(c_j) - \hat{p}_j(c_j)| < \varepsilon/3$, which also implies $\| \hat{p}_j^n - \hat{p}_j \| < \varepsilon/3$. Since $\hat{\mathfrak{P}}_j$ is dense in $\mathfrak{P}_j$, for any $p_j \in \mathfrak{P}_j$ and $\varepsilon \in (0,\infty)$, there exists $\hat{p}_j \in \hat{\mathfrak{P}}_j$ satisfying $\| \hat{p}_j - p_j \| < \varepsilon/3$. Hence, for any $\varepsilon \in (0,\infty)$, there exists $\bar{n} \in \{1, 2, \cdots\}$ such that $n \geqq \bar{n} \Rightarrow \| p_j^n - p_j \| \leqq \| p_j^n - \hat{p}_j^n \| + \| \hat{p}_j^n - \hat{p}_j \| + \| \hat{p}_j - p_j \| < \varepsilon$. Therefore, the required result is obtained. Further, we define the product subspace $\mathfrak{P} := \mathfrak{P}_1 \times \cdots \times \mathfrak{P}_\ell$ of $L_1(C_1, R) \times \cdots \times L_1(C_\ell, R)$. We denote a typical element of $\mathfrak{P}$ as $p := (p_1, \cdots, p_\ell)$.

2.2 Consumers

We assume that there exist infinitely many consumers. And we consider that they single out a finite ℓ-tuple of desired commodities, each of which belongs to a different category. This implies that there exist no neighboring goods that can be substituted for favorite brands, whichis similar to the hypothesis set up in Hart (1985) and Pascoa (1993). We further suppose that their favorite commodity vectors differ, thus, we can identify a consumer by his or her desired differentiated commodities. Accordingly, every consumer makes consumption plans for his or her favorite brands $c_1, \cdots, c_\ell$ and the homogeneous commodity $c_{\ell+1}$. For a consumer choosing $c \in C$, let $X(c)$, $e(c)$ and $\succsim_c$ be the set of all possible consumption plans, an initial endowment, and a preference relation defined on $X(c)$. Suppose that $\mathbb{B}(R)$ is the collection of Borel subsets of R, and $\mathbb{B}$ denotes the $\ell+1$-fold products $\mathbb{B}(R) \otimes \cdots \otimes \mathbb{B}(R)$. Then, we set up the following assumption based on those concepts.

(A. 1) (i) The graph of $X : C \twoheadrightarrow R^{\ell+1}$ is a member of $\mathbb{C} \otimes \mathbb{B}$;
(ii) $X(c)$ is nonempty, closed and convex for almost every $c \in C$;
(iii) $\succsim_c$ is reflexive, complete, transitive and strictly convex for almost every $c \in C$;
(iv) $\{(x, y) \in X(c) \times X(c) | x \succsim_c x'\}$ is closed in $R^{\ell+1} \times R^{\ell+1}$ for almost every $c \in C$;
(v) $\{c \in C | x(c) \succsim_c x'(c)\} \in \mathbb{C}$ for any measurable selection x and x' from X;

(vi) $e : C \rightarrow R^{\ell+1}$ is a measurable function which satisfies $e(c) := (0, \cdots, 0, e_{h+1}(c), \cdots, e_{\ell+1}(c))$ and $e_{h+1}(c) < \infty, \cdots, e_{\ell+1}(c) < \infty$ for almost every $c \in C$.

We are already familiar with the above assumptions, except for the strict convexty of $\succsim_c$ in (ii) and the first condition in (vi). We reluctantly set up the strong supposition on $\succsim_c$ to study price setting behavior by firms, based on single-valued demand functions. The first assumption in (vi) means that almost every consumer does not initially hold commodities produced by firms. We require this condition simply to discuss the behavior of firms.

To define an individual demend function for each consumer, we must at least make mention of production plans of the firms. For any $j \in \{1, \cdots, \ell\}$, we describe the j-th component of a production plan for a firm which produces commodity $c_j \in C_j$, or uses it as a production factor as $y_j(c_j)$. As we explain in the following section, since every c_j is produced or used as a production factor by only one firm, $y_j(c_j)$ is also a total output or input. Let $L_\infty(C_j)$ be a set defined by $\{y_j : C_j \rightarrow R \mid y_j$ is measurable and $\|y_j\|_\infty < \infty\}$, in which $\|y_j\|_\infty := \inf\{M \geqq 0 \mid \|y_j(c_j)\| \leqq M$ holds for almost every $c_j \in C_j\}$. And then, we consider a nonempty subset $\mathfrak{Y}_j$ of $L_\infty(C_j)$ as the whole set of all outputs or inputs for commodities in the j-th category. We provisionally assume that $L_\infty(C_j)$ has the weak* topology and that $\mathfrak{Y}_j$ is compact in this topology. We denote a total amount of $\ell+1$-th homogeneous commodity as $y_{\ell+1}$. We tentatively hypothesize that $y_{\ell+1}$ is an element of a compact subset $\mathfrak{Y}_{\ell+1}$ of R. In the following section, we define these concepts exactly. Furthermore, we define a product subspace $\mathfrak{Y} := \mathfrak{Y}_1 \times \cdots \times \mathfrak{Y}_{\ell+1}$ of $L_\infty(C_1) \times \cdots \times L_\infty(C_\ell) \times R$, and denote an element of $\mathfrak{Y}$ by $y := (y_1, \cdots, y_{\ell+1})$. As p and y are given, we describe a dividend for consumer c by $w(c, p, y)$, which should be nonnegative. For the time being, we assume that $w(c, \cdot, \cdot) : \mathfrak{P} \times \mathfrak{Y} \rightarrow R$ is continuous, and that $w(\cdot, p, y) : C \rightarrow R$ is measurable. In the following section, we give the precise definition of the function w and prove that its properties may be naturally derived. In consideration of the above, a budget set for each consumer c is defined by $B(c, q, p, y) := \{x \in X(c) \mid \sum_{j=1}^{\ell} q_j x_j + x_{\ell+1} \leqq \sum_{j=1}^{\ell} q_j e_j(c) + e_{\ell+1}(c) + w(c, p, y)\}$. Consequently, an individual demand function $x^* : C \times \prod_{j=1}^{\ell}[a_j, b_j] \times \mathfrak{P} \times \mathfrak{Y} \rightarrow R^{\ell+1}$ is defined by $x^*(c, q, p, y) := x^* \in B(c, q, p, y)$ satisfying $(\forall x \in B(c, q, p, y), x^* \succsim_c x)$.

Now that we have the exact definition of the individual demand function x^*, we can turn to a study of the total demand function for each commodity c_j. If μ is a measure on $\mathbb{C}$, which isabsolutely continuous with respect to σ, then there is a finite measurable function ϕ satisfying $\mu(E) = \int_E \phi d\sigma$ for any $E \in \mathbb{C}$ by Radon-Nikodym theorem (for example, Theorem B in Halmos (1950, p. 128)). Let C_{-j} be the product set defined by $C_1 \times \cdots \times C_{j-1} \times C_{j+1} \times \cdots \times C_\ell$ and $\mathbb{C}_{-j}$ be the product σ-algebra defined on C_{-j}. If σ_{-j} is the product measure on $\mathbb{C}_{-j}$, then we can define a conditional probability of $A := E_1 \times \cdots \times E_{j-1} \times C_j \times E_{j+1} \times \cdots \times E_\ell$ given c_j, or, more precisely, $C_1 \times \cdots \times C_{j-1} \times \{c_j\} \times C_{j+1} \times \cdots \times C_\ell$ as $\mu(A| c_j) := (\int_{E-j} \phi d\sigma_{-j})(\int_{C-j} \phi d\sigma_{-j})^{-1}$. Conventionally, we consider that $\mu(\cdot \mid c_j)$ is defined by $\mu(A| c_j) = 0$ for any $A \in \mathbb{C}$, in the case of $\int_{C-j} \phi d\sigma_{-j} = 0$. If x^*_j is the j-th coordinate function of x^*, then the total demand functions $\xi_j : C_j \times \mathfrak{P}_1 \times \cdots \times [a_j, b_j] \times \cdots \times \mathfrak{P}_\ell \times \mathfrak{Y} \rightarrow R$ and $\xi_{\ell+1} : \mathfrak{P} \times \mathfrak{Y} \rightarrow R$ are defined as:

$$\xi_j(c_j, p_1, \cdots, q_j, \cdots, p_\ell, p, y) := \int_C x_j^*(\cdot, p_1, \cdots, q_j, \cdots, p_\ell, p, y) d\mu(\cdot \mid c_j), \tag{2}$$

$$\xi_{\ell+1}(p, y) := \int_C x_{\ell+1}^*(\cdot, p(\cdot), p, y) d\mu. \tag{3}$$

Lemma 1. (i) $\xi_j (c_j, \cdots\cdot,) : \mathfrak{P}_1 \times \cdots \times [a_j, b_j] \times \cdots \times \mathfrak{P}_\ell \times \mathfrak{P} \times \mathfrak{Y} \rightarrow$ *R is continuous for any* $j \in \{1, \cdots, \ell\}$ *and almost every* $c_j \in C_j$,
(ii) $\xi_{\ell+1} : \mathfrak{P} \times \mathfrak{Y} \rightarrow R$ *is also continuous,*
(iii) $\xi_j (\cdot, p, y) : C_j \rightarrow R$ *is measurable for any* $j \in \{1, \cdots, \ell\}$, $p \in \mathfrak{P}$ *and* $y \in \mathfrak{Y}$.

Proof. (i) Fix $j \in \{1, \cdots, \ell\}$ arbitrarily. For almost every $c \in C$, since we assume that $w(c, \cdot, \cdot)$ is continuous, $B(c, \cdot, \cdot, \cdot)$ is a continuous correspondence having nonempty, convex, and compact values under (A. 1)(ii) and (vi). Therefore, under (A. 1)(iii) and (iv), it is shown by standard methods that $x_j^* (c, \cdot, \cdot, \cdot) : \prod_{j=1}^{\ell}[a_j, b_j] \times \mathfrak{P} \times \mathfrak{Y} \rightarrow R$ is continuous for almost every $c \in C$. Further, $w(\cdot, p, y)$ is measurable for any $p \in \mathfrak{P}$ and $y \in \mathfrak{Y}$ by our provisional supposition. Thus, under the assumptions (A. 1)(i), (v), and (vi), Theorem 17. 18 in Aliprantis and Border (1994, p. 570) verifies that the function $x_j^*(\cdot, p_1, \cdots, q_j, \cdots, p_\ell, p, y) : C \to R$ is measurable for any $p \in \mathfrak{P}$, $q_j \in [a_j, b_j]$ and $y \in \mathfrak{Y}$. The condition $p_j^n \to p_j$ in the meaning of $\|\cdot\|$ implies the convergence in measure (for example, Theorem A in Halmos (1950, p. 130)). Hence, it is shown by Theorem 20. 5 (ii) in Billingsley (1995, p. 268) that as $p^n \to p$, $q_j^n \to q_j$ and $y^n \to y$, the sequence $\{x_j^*(\cdot, p_1^n, \cdots, q_j^n, \cdots, p_\ell^n, p^n, y^n)\}_{n=1}^{\infty}$ converges in measure to $x_j^*(\cdot, p_1, \cdots, q_j, \cdots, p_\ell, p, y)$. As a result, since $x_j^*(\cdot, p_1, \cdots, q_j, \cdots, p_\ell, p, y)$ is bounded for any $p \in \mathfrak{P}$ and $y \in \mathfrak{Y}$, the required result is obtained from Theorem D in Halmos (1950, p. 110).

(ii) The condition is obtained by the same method used in (i).

(iii) Fix any $j \in \{1, \cdots, \ell\}$, $p \in \mathfrak{P}$ and $y \in \mathfrak{Y}$. Define E_i^n and E_i as follows:

$$E_i^n := \left\{ c \in C \middle| \frac{i-1}{2^n} \leqq x_j^*(c, p(c), p, y) < \frac{i}{2^n} \right\} \quad (i = 1, 2, \cdots, 2^n n), \tag{4}$$

$$E^n := \left\{ c \in C \middle| x_j^*(c, p(c), p, y) \geqq n \right\}. \tag{5}$$

And let $\chi_{E_i^n}$ and χ_{E^n} be the characteristic functions of E_i^n and E^n respectively. By using these notions, a non negative simple function $s^n : C \to R$ may be defined as:

$$s^n(c) := \sum_{i=1}^{n} \frac{i-1}{2^n} \chi_{E_i^n}(c) + n\chi_{E^n}(c). \tag{6}$$

For any $c_j \in C_j$, if we define $E_i^n(c_j) := \{c_{-j} \in C_{-j} \mid c \in E_i^n\}$ $(i = 1, 2, \cdots, 2^n n)$ and $E^n(c_j) := \{c_{-j} \in C_{-j} \mid c \in E^n\}$, then we can obtain the following condition:

$$\begin{aligned}
&\int_C s^n d\mu(\cdot|c_j) \\
&= \sum_{i=1}^{n} \frac{i-1}{2^n} \mu(E_i^n|c_j) + n\mu(E^n|c_j) \qquad (7)\\
&= \left(\int_{C_{-j}} \phi \, d\sigma_{-j}\right)^{-1} \left(\sum_{i=1}^{n} \frac{i-1}{2^n} \int_{E_i^n(c_j)} \phi \, d\sigma_{-j} + n\int_{E^n(c_j)} \phi \, d\sigma_{-j}\right).
\end{aligned}$$

It is shown by the method used in the proof of Theorem B in Halmos (1950, pp. 147 – 148) that the following functions $c_j \mapsto \int_{C_{-j}} \phi d\sigma_{-j}$, $c_j \mapsto \int_{E_i^n(c_j)} \phi d\sigma_{-j}$ and $c_j \mapsto \int_{E^n(c_j)} \phi d\sigma_{-j}$ are measurable. Therefore, $c_j \mapsto \int_C s^n d\mu_{(\cdot \mid c_j)}$ is also measurable. Since the sequence $\{s^n(c)\}_{n=1}^{\infty}$ converges to $x_j^*(c, p(c), p, y)$ for almost every $c \in C$, it follows from Theorem D in Halmos (1950, p. 110) that $\int_C s^n d\mu_{(\cdot \mid c_j)} \to \xi_j\ (c_j, p_1, \cdots, p_j(c_j), \cdots, p_\ell, p, y)$ for any $c_j \in C_j$. Thus, the required result is obtained by Theorem A in Halmos (1950, p. 84). □

2.3 Monopolistically competitive firms

We assume that there exist infinitely many firms. In the same manner as for consumers, firms also single out an ℓ-tuple c of their profitable differentiated commodities, each of which belongs to a different category. We postulate that every firm may be identified with the ℓ-tuple $c \in C$. This means that their profitable differentiated commodity vectors differ. Hence, we can give a firm building a production plan for $c = (c_1, \cdots, c_\ell)$ and the homogeneous commodity $c_{\ell+1}$ the name of firm c. We assume that the set of all operating firms is a nonempty subset J of C. For any $c \in J$, we denote the set of all possible production plans for firm c by $Y(c)$ and we describe a typical element of the set by $\hat{y}(c)$. We set up the following assumption (A. 2).

(A. 2) (i) The graph of $Y : J \twoheadrightarrow R^{\ell+1}$ is a member of $\mathbb{C} \otimes \mathbb{B}$;
(ii) $Y(c)$ is nonempty, closed and convex for almost every $c \in J$;
(iii) $\hat{y}(c) \in Y(c)$ fulfills $\hat{y}_1(c) \geqq 0, \cdots, \hat{y}_h(c) \geqq 0, \hat{y}_{h+1}(c) \leqq 0, \cdots, \hat{y}_{\ell+1}(c) \leqq 0$ for almost every $c \in J$.

The above assumptions (i) and (ii) are standard. Since $\int_C e_j d\mu(\cdot \mid c_j)$ $(j = h + 1, \cdots, \ell)$ and $\int_C e_{\ell+1} d\mu$ are bounded under (A. 1)(vi), $Y(c)$ is indeed compact for any $c \in J$. The condition (iii) is a formal description for the supposition stated in Section 2. 1.

To consider monopolistically competitive situations, we need to specify a property of J. We hypothesize that every differentiated commodity $c_j \in C_j$ is produced or used as a production factor by only one firm $c \in J$. Under this assumption, every market is not under perfect competition, in which an auctioneer adjusts prices, but under monopolistic competition. As a result, every firm can manipulate prices. We introduce a function which associates firm c producing c_j (or using c_j as a production factor) to each commodity c_j, which is denoted as follows:

$$f_j : C_j \to J. \tag{8}$$

It should be noted that c_j is the j-th component of $f_j\ (c_j) = c$. Thus, the property of J stated above implies that f_j is a bijection. Using the function, we define a measure σ_J on the measurable space $(J, \{C \cap J \mid C \in \mathbb{C}\})$ by $\sigma_J := \sigma_j \circ f_j^{-1}$. Furthermore, we can define the composition $Y_j \circ f_j : C_j \to 2^R$. Owing to the correspondence, we can precisely define the set $\mathfrak{Y}_j$, introduced in Section 2. 2, as:

$$\mathfrak{Y}_j := \left\{ y_j \in L_\infty(C_j, R) \middle| y_j(c_j) := \hat{y}_j(f_j(c_j)) \in Y_j(f_j(c_j)),\ \text{a.e.} \right\}. \tag{9}$$

The composition $Y_j \circ f_{j'}$ is defined for any $j \in \{1, \cdots, \ell+1\}$ and $j' \in \{1, \cdots, \ell\}$ since $f_1(C_1) = \cdots = f_\ell(C_\ell) = J$ holds by the definition of f_j . As a matter of convenience, we consider $Y_j \circ f_j$ and $Y_{\ell+1} \circ f_j$ for any $j \in \{1, \cdots, \ell\}$. Thus, $\mathfrak{Y}_{\ell+1}$ is defined as:

$$\mathfrak{Y}_{\ell+1} := \left\{ y_{\ell+1} \in R \middle| y_{\ell+1} := \int_{C_j} \hat{y}_{\ell+1} \circ f_j d\sigma_j,\ \hat{y}_{\ell+1}(f_j(c_j)) \in Y_{\ell+1}(f_j(c_j)),\ \text{a.e.} \right\}. \tag{10}$$

Prior to analyzing price setting behavior by monopolistically competitive firms, we need a formal definition of a dividend, which was introduced in the previous section. We denote a fraction of shares of firm c', owned by consumer c, as $\theta(c, c')$. We set up the following assumption about $\theta(c, c')$.

(A. 3) (i) $\theta : C \times J \to [0, 1]$ is $\mathbb{C} \otimes \mathbb{C}$-measurable;
(ii) The condition $\int_C \theta(\cdot, c')d\sigma = 1$ holds for almost every $c' \in J$.

Now, a dividend $w(c, p, y)$ for consumer c is defined as follows:

$$w(c,p,y) := \max\left\{0, \sum_{j=1}^{\ell} \int_{C_j} \theta(c, f_j(\cdot))p_j y_j d\sigma_j + \int_{C_j} \theta(c, f_j(\cdot))\hat{y}_{\ell+1} \circ f_j d\sigma_j\right\}. \quad (11)$$

For any consumer c, a dividend from firm $c' \in J$ should be defined by $\theta(c, c')(\sum_{j=1}^{\ell} p_j(c')\hat{y}_j(c')+\hat{y}_{\ell+1}(c'))$, in which $p_j(c')$ does not depend on any $c'_{j'}$ $(c'_j \neq c'_{j'})$. And thus, each consumer c ought to receive $\int_J \theta(c, \cdot)(\sum_{j=1}^{\ell} p_j \hat{y}_j + \hat{y}_{\ell+1})d\sigma_J$ as the total dividend. From the above arguments, however, the concept may be also defined by $\sum_{j=1}^{\ell} \int_{C_j} \theta(c, f_j(\cdot))p_j y_j d\sigma_j + \int_{C_j} \theta_{\ell+1}(c, f_j(\cdot))\hat{y}_{+1} \circ f_j d\sigma_j$. For technical reasons, we use the latter notion as the dividend for consumers.

Lemma 2. (i) $w(c, \cdot, \cdot) : \mathfrak{P} \times \mathfrak{Y} \to R$ *is continuous for almost every* $c \in C$;
(ii) $w(\cdot, p, y) : C \to R$ *is measurable for any* $p \in \mathfrak{P}$ *and* $y \in \mathfrak{Y}$.

Proof. (i) It is verified by Corollary 6. 47 in Aliprantis and Border (1994, p. 260) that the function $(p_j, y_j) \mapsto \int_{Cj} p_j y_j d\sigma_j$ is continuous for any $j \in \{1, \cdots, \ell\}$. Furthermore, $\hat{y}_{\ell+1} \circ f_j \mapsto \int_{Cj} \hat{y}_{\ell+1} \circ f_j d\sigma_j$ is also continuous by Theorem D in Halmos (1950, p. 84) since $Y_{\ell+1}(f_j(c_j))$ is compact for almost every $c_j \in C_j$. Thus, the required result may be obtained under (A. 3)(i).

(ii) Under the assumption (A. 3)(i), the property may be proved by the same method used in the proof of Theorem B in Halmos (1950, pp. 147 – 148). □

Therefore, although Lemma 1 was founded on the premise that $w(c,\cdot, \cdot)$ is continuous and $w(\cdot, p, y)$ is measurable, the temporal suppositions may be removed, and the property stated in Lemma 1 is undoubtedly true.

When we attempt to analyze price setting behavior by monopolistically competitive firms, we generally suppose that they recognize net demands or supplies for commodities that they produce or use as production factors. The net demand or supply functions are defined as:

$$\zeta_j(c_j,p_1,\cdots,q_j,\cdots,p_\ell,p,y) := \xi_j(c_j,p_1,\cdots,q_j,\cdots,p_\ell,p,y) - \int_C e_j d\mu(\cdot\,|\,c_j), \tag{12}$$

$$\zeta_{\ell+1}(p,y) := \xi_{\ell+1}(p,y) - \int_C e_{\ell+1} d\mu. \tag{13}$$

As we mentioned in the introductinon, we encounter some difficulties when we examine the behavior by firms. That is, if monopolistically competitive firms do not have adequate production facilities and are, therefore, incapable of satisfying net demands for their products, then they cannot charge prices along the demand curves for their products. If such an occasion arises, we presume that the firms supply their products to the best of their ability. For any $j \in \{1, \cdots, h\}$, suppose that D_j is a subset of $C_j \times \mathfrak{P}_1 \times \cdots \times [a_j, b_j] \times \cdots \times \mathfrak{P}_\ell \times \mathfrak{P} \times \mathfrak{Y}$ defined as:

$$D_j := \{(c_j,p_1,\cdots,q_j,\cdots,p_\ell,p,y) | \zeta_j(c_j,p_1,\cdots,q_j,\cdots,p_\ell,p,y) \leqq \bar{y}_j(c_j)\}, \tag{14}$$

in which $\bar{y}_j\ (c_j)$ is the upper bound of the j-th component $Y_j\ (f_j\ (c_j))$ of $Y\ (f_j\ (c_j))$. Then, we may define a modified net demand function $\zeta^*_j : C_j \times \mathfrak{P}_1 \times \cdots \times [a_j, b_j] \times \cdots \times \mathfrak{P}_\ell \times \mathfrak{P} \times \mathfrak{Y} \rightarrow R$ as:

$$\zeta^*_j(c_j,p_1,\cdots,q_j,\cdots,p_\ell,p,y) := \begin{cases} \zeta_j(c_j,p_1,\cdots,q_j,\cdots,p_\ell,p,y) & \text{on } D_j \\ \bar{y}_j(c_j) & \text{on } D_j^C. \end{cases} \tag{15}$$

Note that ζ^*_j is equal to ζ_j for any $j \in \{h+1 \cdots, \ell\}$ since we have no problem about production factors. It is clear by Lemma 1 that $\zeta^*_j(c_j, \cdot, \cdots, \cdot) : \mathfrak{P}_1 \times \cdots \times [a_j, b_j] \times \cdots \times \mathfrak{P}_\ell \times \mathfrak{P} \times \mathfrak{Y} \rightarrow R$ is continuous and $\zeta^*_j(\cdot, p, y) : C_j \rightarrow R$ is measurable. By using the modified net demend functions, we can define $\pi : J \times \prod_{j=1}^{\ell}[a_j, b_j] \times \mathfrak{P} \times \mathfrak{Y} \rightarrow R$ as:

$$\pi(c,q,p,y) := \sum_{j=1}^{\ell} q_j \zeta^*_j(c_j,p_1,\cdots,q_j,\cdots,p_\ell,p,y) + \zeta^*_{\ell+1}(p,y). \tag{16}$$

Thus, we can define a profit function $\pi^* : J \times \mathfrak{P} \times \mathfrak{Y} \rightarrow R$ as:

$$\pi^*(c,p,y) := \max\left\{\pi(c,q,p,y)\,\middle|\, q \in \prod_{j=1}^{\ell}[a_j,b_j]\right\}. \tag{17}$$

Furthermore, we define a correspondence $Q : J \times \mathfrak{P} \times \mathfrak{Y} \twoheadrightarrow R^\ell$ associating the most profitable price vectors to (c, p, y) as:

$$Q(c,p,y) := \left\{q^* \in \prod_{j=1}^{\ell}[a_j,b_j]\,\middle|\, \pi^*(c,p,\eta) = \pi(c,q^*,p,y)\right\}. \tag{18}$$

We set up the following assumption about the map $q \mapsto \pi(c, q, p, y)$.

(A. 4) For almost every $c \in J$, and any $p \in \mathfrak{P}$ and $y \in \mathfrak{Y}$, $q \mapsto \pi(c, q, p, y)$ is quasi concave.

The assumption is strong, however, we need it in order to guarantee that the correspondence Q has convex values.

Lemma 3. (i) $Q(c, \cdot, \cdot) : \mathfrak{P} \times \mathfrak{Y} \twoheadrightarrow R^\ell$ is an upper semicontinuous correspondence with nonempty and convex values for almost every $c \in J$;
(ii) The graph of $Q(\cdot, p, y) : J \twoheadrightarrow R^\ell$ is a member of $\mathbb{C} \otimes \mathbb{B}$ for any $p \in \mathfrak{P}$ and $y \in \mathfrak{Y}$.

Proof. (i) (Nonempty-valuedness) For any $j \in \{1, \cdots, \ell\}$, it was proved in Section 2. 1 that $\mathfrak{P}_j$ is compact for the norm $\|\cdot\|$. It is shown by Alaoglu's Theorem (for example, Theorem 6. 25 in Aliprantis and Border (1994, p. 250)) that $\mathfrak{Y}_j$ is weakly* compact for any $j \in \{1, \cdots, \ell + 1\}$. Therefore, the domain $\prod_{j=1}^{\ell}[a_j, b_j] \times \mathfrak{P} \times \mathfrak{Y}$ of $\pi(c, \cdot, \cdot, \cdot)$ is compact for the product topology. Since the function is continuous by Lemma 1 (i) and (ii), the required condition is obtained. (Convex-valuedness) The condition is also clear by (A. 4). (Upper semicontinuity) The condition may be shown by the theorem in Berge (1963, p. 116) since $\pi(c, \cdot, \cdot, \cdot)$ is continuous.

(ii) The required condition is obtained from Theorem 17. 18 in Aliprantis and Border (1994, p. 570) under the result of Lemma 1 (iii). □

By using the correspondence Q and the function f_j, we define a price information correspondence $P_j : C_j \times \mathfrak{P} \times \mathfrak{Y} \twoheadrightarrow R$ for commodities belonging to the j-th category as:

$$P_j(c_j, p, y) := Q_j(f_j(c_j), p, y). \tag{19}$$

It is clear from Lemma 3 that $P_j(c_j, \cdot , \cdot) : \mathfrak{P} \times \mathfrak{Y} \twoheadrightarrow R$ is upper semi continuous and that the graph of $P_j (\cdot , p, y) : C_j \twoheadrightarrow R$ is a member of $\mathbb{C}_j \otimes \mathbb{B}(R)$.

3 Existence of equilibria for a monopolistically competitive economy

3.1 Definition of a monopolistically competitive economy

We formally define a monopolistically competitive economy as:

(D. 1) $E_{mono} := \{X, e, \succsim, Y, \theta\}$.

We define an equilibrium for E_{mono} as follows:

(D. 2) *An equilibrium for* E_{mono} *is* (p^*, x^*, y^*) *fulfilling the following conditions* (i) — (iii):
(i) *Almost every consumer* $c \in C$ *selects the consumption plan* $x^*(c) \in B(c, p^*(c), p^*, y^*)$ *satisfying the condition* $(\forall x \in B(c, p^*(c), p^*, y^*), x^*(c) \succsim_c x)$;
(ii) *Almost every firm* $c \in J$ *chooses the prices* $(p_1^*(c_1), \cdots, p_\ell^*(c_\ell)) \in \prod_{j=1}^{\ell}[a_j, b_j]$ *satisfying* $\pi^*(c, p^*, y^*) = \sum_{j=1}^{\ell} p_j^*(c_j)\zeta_j(c_j, p_1^*, \cdots, p_j^*(c_j), \cdots, p_\ell^*, y^*) + \zeta_{\ell+1}(p^*, y^*)$;
(iii) *For any* $j \in \{1, \cdots, \ell\}$ *and almost every commoditiy* $c_j \in C_j$, $\zeta_j(c_j, p_1^*, \cdots, p_j^*(c_j), \cdots, p_\ell^*, y^*) = y_j^*(c_j, p^*, y^*)$, *and for the* $\ell+1$*-th commodity,* $\zeta_{\ell+1}(p^*, y^*) = y_{j+1}^*$.

3.2 Theorem on the existence of equilibria for a monopolistically competitive economy

In this section, we prove the existence of equilibria for the economy E_{mono}. To establish an existence theorem on the equilibria for the economy, we set up the following assumption (A. 5).

(A. 5) For any $j \in \{1, \cdots, h\}$, $q_j \in [a_j, b_j]$, $q'_j \in [a_j, b_j]$, $p \in \mathfrak{P}$, $y \in \mathfrak{Y}$ and almost every $c_j \in C_j$, $q_j < q'_j \Rightarrow \zeta_j(c_j, p_1, \cdots, q'_j, \cdots, p_\ell, p, y) < \zeta_j(c_j, p_1, \cdots, q_j, \cdots, p_\ell, p, y)$.

The assumption means that the function ζ_j is decreasing with respect to q_j, and hence, what is called the total income effect should be sufficiently small.

Theorem. *There exist equilibria for the economy E_{mono} under* (A. 1) — (A. 5).

Proof. (I) For any $j \in \{1, \cdots, \ell\}$, let $\alpha_j : \mathfrak{P} \times \mathfrak{Y} \twoheadrightarrow \mathfrak{P}_j$ be a correspondence defined as:

$$\alpha_j(p, y) := \{p_j \in \mathfrak{P}_j | p_j(c_j) \in P_j(c_j, p, y),\ a.e.\}. \tag{20}$$

It is shown by Lemma 3 and Theorem 8. 1. 3 in Aubin and Frankowska (1990, p. 308) that α_j is nonempty-valued. It is clear by Lemma 3 (i) that the correspondence is convex-valued. And, it follows from Lemma 3 and Theorem D in Halmos (1950, p. 110), that the correspondence has a closed graph. Suppose that $\alpha : \mathfrak{P} \times \mathfrak{Y} \twoheadrightarrow \mathfrak{P}$ is defined as follows:

$$\alpha(p, y) := \alpha_1(p, y) \times \cdots \times \alpha_\ell(p, y), \tag{21}$$

then it is quite clear that the correspondence α also has such properties.

(II) For any $j \in \{1, \cdots, \ell\}$, let $\beta_j : \mathfrak{P} \times \mathfrak{Y} \twoheadrightarrow \mathfrak{P}_j$ be a correspondence defined as follows:

$$\beta_j(p, y) := \{y_j^*(\cdot, p, y)\};\ (y_j^*(c_j, p, y) = \zeta_j^*(c_j, p, y),\ a.e.) \tag{22}$$

It is clear by the definition of ζ_j^* that β_j is nonempty-valued. Since $\beta_j(p, y)$ is a singleton, it is clearly convex-valued. We turn to prove that the correspondence has a closed graph. It may be shown that the sequence $\{\zeta_j^*(\cdot, p^n, y^n)\}_{n=1}^\infty$ converges weakly* to $\zeta_j^*(\cdot\, p, y)$ as $(p^n, y^n) \rightarrow (p, y)$. Let $\{F^1, \cdots, F^m\}$ be a partition of C_j and $\{a^1, \cdots, a^m\}$ be a set of real numbers, such that $\hat{p}_j(c_j) = a^h$ if $c_j \in F^h$ $(h = 1,$

$\cdots,m)$; $\hat{p}_j(c_j) = 0$ if $c_j \notin F^1 \cup \cdots \cup F^m$. And, let $\hat{p}_j(c_j)$ be a simple function $\sum_{h=1}^{m} a^h \chi_{F^h}(c_j)$, in which χ_{F^h} is a characteristic function of F^h. It follows from Lemma 1 and Theorem 20. 5 in Billingsley (1995, p. 268) that $\{\zeta_j^*(\cdot, p^n, y^n)\}_{n=1}^{\infty}$ converges in measure to $\zeta_j^*(\cdot\, p, y)$ as $p^n \to p$ and $y^n \to y$. Thus, the following condition holds by Theorem D in Halmos (1950, p. 110) for any $\hat{p}_j$ in a strongly dense subset of $\mathfrak{P}_j$:

$$\begin{aligned}
&\lim_{n\to\infty}\int_{C_j} \hat{p}_j\zeta_j^*(\cdot, p^n, y^n)d\sigma_j \\
&\qquad = \lim_{n\to\infty}\int_{C_j}\sum_{h=1}^{m} a^h\chi_{F^h}\zeta_j^*(\cdot, p^n, y^n)d\sigma_j \\
&\qquad = \sum_{h=1}^{m} a^h \lim_{n\to\infty}\int_{F^h}\zeta_j^*(\cdot, p^n, y^n)d\sigma_j \\
&\qquad = \sum_{h=1}^{m} a^h \int_{F^h}\zeta_j^*(\cdot, p, y)d\sigma_j \\
&\qquad = \int_{C_j}\sum_{h=1}^{m} a^h\chi_{F^h}\zeta_j^*(\cdot, p, y)d\sigma_j \\
&\qquad = \int_{C_j}\hat{p}_j\zeta_j^*(\cdot, p, y)d\sigma_j.
\end{aligned} \tag{23}$$

Therefore, it follows from Theorem 10 in Yosida (1980, p. 125) that $\{\zeta_j^*(\cdot, p^n, y^n)\}_{n=1}^{\infty}$ converges weakly* to $\zeta_j^*(\cdot, p^n, y^n)$. Suppose that $\{y_j^*(\cdot, p^n, y^n)\}_{n=1}^{\infty}$ is a sequence of $\mathfrak{Y}_j$ satisfying the condition $\zeta_j^*(\cdot, p^n, y^n) = y_j^*(\cdot, p^n, y^n)$ for any $n \in \{1, 2, \cdots\}$. Since $\mathfrak{Y}_j$ is compact in the weak* topology, $\{y_j^*(\cdot, p^n, y^n)\}_{n=1}^{\infty}$ also converges weakly* to $y_j^*(\cdot, p, y)$ and $\zeta_j^*(\cdot, p, y) = y_j^*(\cdot, p, y) \in \mathfrak{Y}_j$. Thus, β_j has a closed graph. Let $\beta : \mathfrak{P} \times \mathfrak{Y} \twoheadrightarrow \mathfrak{Y}$ be a correspondence defined as follows:

$$\beta(p, y) := \beta_1(p, y) \times \cdots \times \beta_\ell(p, y). \tag{24}$$

It is clear that β has nonempty and convex values, and its graph is closed.

(III) Define a correspondence $\varphi : \mathfrak{P} \times \mathfrak{Y} \twoheadrightarrow \mathfrak{P} \times \mathfrak{Y}$ by $\varphi(p, y) := \alpha(p, y) \times \beta(p, y)$. It follows from Fan-Glicksberg's fixed point theorem (Fan (1952) and Glicksberg (1952)) that φ has a fixed point $(p^*, y^*) \in \varphi(p^*, y^*)$.

(IV) It is clear that the condition (i) in (D. 2) is satisfied at $(p, y) \in \mathfrak{P} \times \mathfrak{Y}$. We claim that the properties (ii) and (iii) in (D. 2) are also true. To see this, we must show that the condition $\zeta_j(c_j, p^*, y^*) = \zeta_j^*(c_j, p^n, y^n) = y_j^*(c_j, p^*, y^*) \leqq \bar{y}_j(c_j)$ holds for almost every $c_j \in C_j$. Define $\hat{C}_j := \{c_j \in C_j \mid \zeta_j(c_j, p^*, y^*) > \bar{y}_j(c_j)\}$. Suppose, on the contrary, that $\sigma_j(\hat{C}_j) > 0$ holds for some $j \in \{1, \cdots, \ell\}$. Then, the value of the j-th component of $\pi^*(f_j(c_j), p^*, y^*)$ ought to be $p_j^*(c_j)\bar{y}_j(c_j)$ for almost every $c_j \in \hat{C}_j$. By (A. 5), there exists $\hat{p}_j \in \mathfrak{P}_j^n$ satisfying $[\hat{p}_j(c_j) > p_j^*(c_j)$ and $\zeta_j(c_j, p_1^*, \cdots, \hat{p}_j(c_j), \cdots, p_\ell^*, p^*, y^*) = \bar{y}_j(c_j)]$, a. e. in $\hat{C}_j$ for a sufficiently large $n \in \{1, 2, \cdots\}$. Thus, $\hat{p}_j(c_j)\zeta_j(c_j, p_1, \cdots, \hat{p}_j(c_j), \cdots, p_\ell, p, y) = \hat{p}_j(c_j)\bar{y}_j(c_j) > p_j^*(c_j)\bar{y}_j(c_j)$ holds, a. e. in $\hat{C}_j$, however, this is inconsistent with the property of the most profitable price p_j^*. Hence, the required condition is obtained, and $\sum_{j=1}^{\ell} p_j^*(c_j)\zeta_j^*(c_j, p^*, y^*) + \zeta_{\ell+1}^*(p^*, y^*)$ is equal to $\sum_{j=1}^{\ell} p_j^*(c_j)\zeta_j(c_j, p^*, y^*) + \zeta_{\ell+1}(p^*, y^*)$. Accordingly, the condition (ii) in (D. 2) is satisfied. Further, from the above argument, $\zeta_j^*(c_j, p^*, y^*) = y_j^*(c_j, p^*, y^*)$ holds for almost every $c_j \in C_j$. And thus, $\zeta_{\ell+1}(p^*, y^*) = y_{\ell+1}^*$ also follows from the definition of budget constraints for consumers and (A. 3)(ii). Therefore, the condition (iii) in (D. 2) is satisfied. □

References

ALIPRANTIS, C. D., AND K. C. BORDER (1994): *Infinite Dimensional Amalysis, A Hitchhiker's Guide 2nd Edition*. Springer-Verlag.

ARROW, K. J., AND F. H. HAHN (1971): *General competitive analysis*. Holdenday, San Francisco.

AUBIN, J.-P., AND H. FRANKOWSKA (1990): *Set-valued analysis*. Birkhauser.

BENASSY, J. P. (1988): "The objective demand curve in general equilibrium with price makers," *Economic Journal*, 98, 37–49.

BERGE, C. (1963): *Topological Spaces: including a treatment of multi-valued function, vector spaces, and convexity*. Oliver and Boyd, Edinburgh and London, (A translation of the work first published by Dunod, Paris, in 1959 under the title *Espaces de topologiques, fonctions multivoques.)*.

BILLINGSLEY, P. (1995): *Probability and Measure, Third edition, Wiley Series in Probability and Mathematical Statistics*. John Wiley and Sons.

Codognato, G., and J. J. Gabszewicz (1993): "Cournot-Walras equilibria in markets with a continuum of traders," *Economic Theory*, 3, 453–464.

Cornwall, R. (1977): "The concept of general equilibrium in a market economy with imperfectly competitive producers," *Metroeconomica*, 29, 57–72.

Fan, K. (1952): "Fixed points and minimaxtheorems in locally convex linear spaces," *Proceedings of the National Academy of Sciences, USA*, 38, 121–126.

Fitzroy, F. (1974): "Monopolistic equilibrium, non-convexity and inverse demand," *Journal of Economic Theory*, 7, 1–16.

Gabszewicz, J. J., and J. P. Vial (1972): "Oligopoly 'A la Cournot' in general equilibrium analysis," *Journal of Economic Theory*, 4, 381–400.

Glicksberg, I. L. (1952): "A further generalization of the Kakutani fixed point theorem with applications to Nash equilibrium points," *Proceedings of the American Mathematical Society*, 3, 170–174.

Halmos, P. R. (1950): *Measure Theory*. Springer-Verlag.

Hart, O. D. (1979): "Monopolistic competition in a large economy with differentiated commodities," *Review of Economic Studies*, 46, 1–30.

—— (1985): "Monopolistic competition in the spirit of Chamberlin: A general model," *Review of Economic Studies*, 52, 529–546.

Laffont, J. J., and G. Laroque (1976): "Existence d'un équilibre général de concurrence imparfaite: une introduction," *Econometrica*, 44, 283–294.

Marschak, T., and R. Selten (1974): *General equilibrium with price making firms, Lecture Notes in Economics and Mathematical Systems, Vol.580*. Springer-Verlag, Berlin.

Negishi, T. (1961): "Monopolistic competition and general equilibrium," *Review of Economic Studies*, 28, 196–201.

Pascoa, M. R. (1993): "Noncooperative equilibrium and Chemberlinian monopolistic competition," *Journal of Economic Theory*, 60, 335–353.

Silvestre, J. (1977a): "General monopolistic equilibrium under non-convexities," *International Economic Review*, 18, 425–434.

—— (1977b): "A model of general equilibrium with monopolistic behavior," *Journal of Economic Theory*, 16, 425–442.

Yosida, K. (1980): *Functional Analysis, Sixth Edition*. Springer-Verlag.