Kansai University Review of Economics
No. 10 (March 2008), pp. 1–23

Search Friction and Job Destruction under Decreasing Marginal Return Technology

Hiromi Nosaka*†

This paper considers the role of job destruction in a job search model where firms have decreasing marginal return technology and workers are heterogeneous. Because the bargaining solution developed by Stole and Zwiebel (1996a) and Smith (1999) is employed, firms have an incentive to overemploy workers so as to reduce the workers' bargaining position. This creates a discrepancy between the two efficiency conditions: namely, job creation and job destruction. Consequently, any equilibrium is unable to attain an efficient allocation when heterogeneous workers exist within a firm.

Keywords: search frictions, bargaining, heterogeneous workers

1 Introduction

Search frictions are considered to play important roles in characterizing labor markets (Mortensen and Pissarides, 1994). Although the literature has been successful in understanding many stylized facts of the dynamics of unemployment (Merz, 1995; Andolfatto, 1996; Cooley and Quadrini, 1999; Den Haan et al., 2000; Walsh, 2003; Trigari, 2004), firm structure as modeled had been portrayed as relatively simple until recently in the sense that there is no interaction between workers within a firm. This is because most models assume that a firm employs up to only one worker.

Pissarides (2000) and Cahuc and Wasmer (2001) showed that this assumption is not restrictive, when the firm uses constant returns-to-

* Faculty of Economics, Kansai University, 3-3-35 Yamate-cho, Suita, Osaka, 564-8680, Japan. Phone: +816-6368-0094. Email: *hnosaka@ipcku.kansai-u.ac.jp*

† I would like to thank seminar participants at the Hong Kong University of Science and Technology for their helpful comments. All remaining errors are mine. Financial support from the JSPS (Grant-in-Aid for Scientific Research (C) 19530224) is also gratefully acknowledged.

scale technology with respect to labor (L) and capital (K) and where capital can be adjusted freely in the competitive capital market. In this case, production is expressed as $Y=F(K,L)=F(k,1)\ L$, where the capital labor ratio, k, depends only on the market rental rate of capital.[1] Then, due to the linearity of the technology, one firm employing L workers is equivalent to L firms, each of which employs one worker.[2] Although this type of model is simple and tractable, it omits many of the more important interactions within the firm. For example, when there are adjustment costs in changing capital, the marginal product of labor is a decreasing function of labor input in the short run. Another example is the case of the monopolistic competition, where marginal revenue is decreasing in labor input (Blanchard and Kiyotaki, 1987; Rotemberg, 2006; Krause and Lubik, 2007).

Stole and Zwiebel (1996a, 1996b) proposed a bargaining solution in static models under such situations.[3] This is a natural extension of the Nash bargaining solution with many agents where the decreasing marginal return technology is critical. They obtained a simple, closed-form solution, which can be applied to many economic situations. They further showed that the equilibrium allocation is never efficient because the firm has an incentive to overemploy workers. By employing a greater number of workers, the marginal productivity of each worker falls under the decreasing marginal return technology, thereby reducing the contribution of each worker to production. As a consequence, the firm can reduce the bargaining power of workers by employing more workers than the socially optimal level.

Although they focus on partial equilibrium models without labor markets outside the firm, the general equilibrium aspects of these models were also explored by Smith (1999), Cahuc and Wasmer (2001), and Cahuc et al. (2004).[4] These studies introduced a search

---

1 From the first-order condition of capital, $R=F_K(K,L)=F_K(k,1)$, where R is the rental rate of capital.

2 Marimon and Zilibotti (2000) avoided the issue of a bargaining solution with decreasing marginal returns by introducing two sectors: the first is the sector for final goods where firms face decreasing marginal revenue, and the second is the sector for intermediate goods where the production function is linear in labor input. Under this decentralized version of the economy, employed workers only bargain over wages with the producers of intermediate goods whose technology is linear. Consequently, any complexity in the bargaining problem is missing.

3 Wolinsky (1996, 2000) considered the dynamic implications of this solution where the firm can adjust employment over time and showed that a variety of equilibria exist.

4 De Fontenay and Gans (2003) showed that firms underemploy workers when some external labor pool is

friction in the labor market to explicitly distinguish between unemployed and employed workers. Smith (1999) showed that equilibrium allocation is never efficient, because it is distorted by the incentives of the firms to overemploy workers. In Smith's (1999) general equilibrium model, the firm makes two decisions: first, the optimal choice of job creation; second, the decision to enter the market. Smith (1999) showed that these decisions never lead to an efficient allocation in equilibrium. Under the decreasing marginal return technology, the firm has an incentive to overemploy workers, and, consequently, efficiency requires a low bargaining share of the firm so as to make job creation less profitable. This adjustment, however, reduces the incentive of the firm to enter the market, because profit is low when its bargaining share is small. Smith (1999) showed there is no level of bargaining share that leads to an efficient allocation.

Although these studies are clearly suggestive, the workers are homogeneous in their models, and, as a consequence, job separation does not endogenously occur for workers whose productivity is relatively lower. The current study proposes a dynamic general equilibrium model with search frictions and heterogeneous workers, where the decision on job separation is critical. As noted, the previous literature investigates the efficient numbers of firms and the amount of job creation but not of job destruction in an environment where the decreasing marginal return technology is critical.

A closed-form solution that includes heterogeneous workers is provided as an extension of Stole and Zwiebel (1996a), Smith (1999), Cahuc and Wasmer (2001) and Bertola and Garibaldi (2001). Accordingly, this solution preserves similar properties to those of the standard job search models with homogeneous workers — including the Nash bargaining solution and Shapley value as special cases. The analysis provides a simple setup where the bargaining outcome in the literature can be applied in the case of heterogeneous workers.

The present paper also reveals that efficient allocation is never achieved in equilibrium when heterogeneous workers exist because it requires an additional condition for job separation. As noted, Stole and

---

available during the bargaining process, and consequently, the welfare results depend on whether the external labor pool is available.

Zwiebel (1996a) and Smith (1999) did not take the decision of job separation into consideration. In order to portray the relevance of heterogeneous workers, we first consider the case of homogeneous workers as the benchmark case. Under the assumptions in the present paper, efficient allocation is possible for some appropriate choice of bargaining share because it makes the level of job creation efficient in equilibrium.[5] In the case of heterogeneous workers, however, it is impossible to attain efficiency in equilibrium. This is because there are two distinct conditions for efficient job creation and job destruction when workers are heterogeneous. It is true that these two conditions are satisfied simultaneously in standard job search models without decreasing marginal return technology. As a result, efficiency can be attained if bargaining power is chosen appropriately. However, the introduction of decreasing marginal return technology and intrafirm bargaining violates this efficiency result.

The current work is closely related to job search models, particularly those including the heterogeneity of jobs and endogenous job separation. Heterogeneity of jobs has been extensively studied in the job search models (Davis, 2001; Acemoglu, 2001; Cahuc et al., 2004). When firms create productive job openings (good jobs), they are unable to reap the benefits fully because parts of the benefits go to workers through negotiation after employment. They show that when the types of jobs are heterogeneous, a random matching model leads to an inefficient allocation in which there are too many job openings for less-productive firms (bad jobs) in equilibrium. These results are clearly related to the present model. However, these models do not consider the effects of job separation, and this is the focus of the current paper. Moreover, we assume that workers (and hence, jobs) are identical when they are hired, and the quality of jobs is homogeneous in opening jobs. After they are employed, an exogenous productivity shock arrives for individual workers and endogenous job separation arises.

The present paper clearly belongs to the job search literature with endogenous job destruction (Bertola and Caballero, 1994; Bertola and Garibaldi, 2001; Koeniger and Prat, 2007). Bertola and Caballero

---

[5] The decision on market entry, a critical factor in Smith (1999), is excluded so as to clarify the effect on the job separation decision.

(1994) showed that the market equilibrium is unable to attain a social optimum when the productivity of firms is heterogeneous in job search models with decreasing marginal return technology. Cross-sectional differences in productivity across firms requires the adjustment of the behavior of individual firms differently in order to induce efficient job vacancies, but this is generally impossible. These models are clearly suggestive, but do not shed light on the role of job separation when there is a heterogeneity of workers within a firm. The present paper, on the other hand, provides a clear condition for efficient job separation, and inefficiency arises in equilibrium, even without heterogeneity across firms.

There are many applications of job search models that employ the bargaining solution in the present paper where decreasing marginal return technology is critical. Bertola and Garibaldi (2001) investigated the effect of firm sizes on wages when the productivity of firms is heterogeneous and showed that wages are higher for larger firms because of the high level of productivity. Koeniger and Prat (2007) examined the effect of employment protection legislation in a model that includes job creation and destruction, along with free entry and exit conditions, although workers are homogeneous. Cahuc et al. (2004) studied the effect of heterogeneous workers and capital, but without endogenous job separation. Kudoh and Sasaki (2007) applied the bargaining solution to study working hours. The bargaining solution is also used in business cycle models such as those of Chéron and Langot (2000), Rotemberg (2006) and Krause and Lubik (2007).

The rest of this paper is organized as follows. Section 2 presents the basic model. Section 3 analyzes the properties of the bargaining solution. Section 4 considers the welfare implications. Section 5 concludes the paper.

2 Model

We assume a small open economy where firms and households can lend and borrow capital with a constant rental rate r, and $\beta=(1+r)^{-1}$ is the corresponding discount rate. We assume a discrete time model with search friction in the labor market. The measure of newly matched workers in each period is determined by the matching technology

$\mu(v^M, u^M)$, where v^M is the measure of total vacancies in the market and u^M is the measure of unemployed workers. We assume that μ is homogeneous of degree one with respect to v^M and u^M. As a result, a worker (denoted by i) can find a job with probability of $p_w = \mu(v^M, u^M)/u^M$. On the other hand, the firm (denoted by j) can find the measure of workers by $p_f = \mu(v^M, u^M)/v^M$ when posting a unit measure of vacancies. By defining the tightness of $\theta = v^M/u^M$, these two are $p_w = q(\theta)\theta$ and $p_f = q(\theta)$, respectively, where $q(\theta) = \mu(1, 1/\theta)$.

2.1 Firms

We assume that a unit measure of firms exists, where each firm is denoted as a point on the unit interval on $[0, 1]$. It is noted that firms cannot enter the market freely, unlike in Smith (1999). One firm employs many workers. Each worker has a weight of Δ for a firm and we take the limiting case of $\Delta \to 0$. Firm j announces the measure of job vacancies at the beginning of the period, which is denoted by v_j.[6] According to the above matching technology, this firm can find $q(\theta)v_j$ measure of potential workers. When firm j announces v_j vacancies, we have the following total vacancies in the market (v^M):

$$v^M = \int_0^1 v_j \, dj. \tag{1}$$

In order to create v_j vacancies, the firm needs to pay the vacancy cost of cv_j. It takes time to employ more workers because of search frictions, but the firm can fire workers immediately. We assume that the workers are fired before production takes place. Let n_j and $\tilde{n}_j$ be the respective measures of employed workers before and after job destruction and these are related according to the following law of motion in employment:

$$\tilde{n}_j \le n_j, \quad n'_j = q(\theta)v_j + (1 - \delta_w)\tilde{n}_j, \tag{2}$$

where δ_w is the job separation rate for exogenous reasons and occurs at the end of each period. Here, n' is the employment level at the

[6] Note that the total number of job openings is v_j / Δ, since each worker has a weight of Δ.

beginning of the next period.

The production function is a sum of two parts: a homogeneous part ($af(\tilde{n}_j)$) and a heterogeneous part (η_i). The homogeneous part is described by the standard production function and depends only on the number of workers ($\tilde{n}_j$) and technology (a). In this expression, the technology of a represents an aggregate shock and we assume that it follows some Markov process. The production function is concave: $f(0)=0$, $f'(\tilde{n})>0$, and $f''(\tilde{n}) \leq 0$.

Workers are heterogeneous in each firm. As a heterogeneous part of production, worker i produces an additional amount of η_i depending on the characteristics of worker i. For simplicity, η_i is drawn from the i.i.d. process across workers and time, and its distribution is $G(\eta)$ with support of $(-\infty, +\infty)$.[7] As is made clear later in the paper, the optimal firing policy of firm j is to fire any workers whose η_i is below some critical value of $\underline{\eta}_j$. Since high-ability workers (workers with high η) remain in the firm, the relation between initial employment (n_j) and employment after job destruction ($\tilde{n}_j$) is:

$$\tilde{n}_j = \bar{G}(\underline{\eta}_j)n_j, \tag{3}$$

where $\bar{G}(\eta)=1-G(\eta)$. With this optimal job destruction rule, the total production of homogeneous and heterogeneous parts is:

$$af(\tilde{n}_j) + n_j \int_{\underline{\eta}_j}^{\infty} \eta \, dG(\eta) = af(\tilde{n}_j) + \tilde{n}_j \Psi(\underline{\eta}_j),$$

where Ψ is the average productivity of the workers after job destruction and

$$\Psi(\underline{\eta}_j) = \frac{\int_{\underline{\eta}_j}^{\infty} \eta \, dG(\eta)}{\bar{G}(\underline{\eta}_j)}.$$

Suppose that the firm, j, whose employment level is n_j, observes a vector of the current aggregate variables of $\mathbf{s}$ at the beginning.[8] We

---

[7] When the heterogeneous part of productivity is correlated over time, we need to include the skill distribution of workers as a state variable in the model.

[8] The shock of a is the only element of $\mathbf{s}$, but many stochastic variables can be included in general.

assume that the firms and the workers decide their decisions only on the current state variables of (n_j, s).[9] Based on this assumption, the firm's value function, denoted as $J(n_j, \mathrm{s})$, is characterized as follows:

$$\begin{aligned} J(n_j, \mathrm{s}) &= \max_{v_j, n'_j, \tilde{n}_j, \underline{\eta}_j} af(\tilde{n}_j) + \tilde{n}_j \Psi(\underline{\eta}_j) - w(\tilde{n}_j, \mathrm{s}, \underline{\eta}_j) \\ &\qquad -cv_j + E_{\mathrm{s}'|\mathrm{s}} J(n'_j, \mathrm{s}')\,\beta, \quad (4) \\ s.t. \quad & n'_j = q(\theta)v_j + (1-\delta_w)\tilde{n}_j, \quad \tilde{n}_j = \bar{G}(\underline{\eta}_j)n_j. \\ & \tilde{n}_j \le n_j. \end{aligned}$$

In the Bellman equation, $w(\tilde{n}_j, \mathrm{s}, \underline{\eta})$ is the total wage that the firm pays all employed workers, and the amount is determined after optimal decisions of $(v_j, n'_j, \tilde{n}_j)$ are made.[10] The wage depends on the number of workers $(\tilde{n}_j)$, the critical quality of each worker $(\underline{\eta}_j)$ as well as other aggregate state variables (s). In order to save on notation, it is convenient to focus on the wage rate of the worker with $\eta_i=0$. Then, by assuming the bargaining solution defined later, we can show that the wage rate for worker i, denoted as $w^r(n, \mathrm{s}, \eta)$, is expressed as follows:

$$w^r(n, \mathrm{s}, \eta) = w^r(n, \mathrm{s}, 0) + \phi\eta, \quad (5)$$

where ϕ is the bargaining power of the workers. Intuitively, since the heterogeneous part of production (η) is additively separable, this part of the flow productivity is shared between the firm and the worker in the standard way. The total wage payment by the firm is:

$$\begin{aligned} w(\tilde{n}_j, \mathrm{s}, \underline{\eta}_j) &= w^r(\tilde{n}_j, \mathrm{s}, 0)\tilde{n}_j + \phi n_j \int_{\underline{\eta}_j}^{\infty} \eta \, dG(\eta), \\ &= w^r(\tilde{n}_j, \mathrm{s}, 0)\tilde{n}_j + \phi\Psi(\underline{\eta}_j)\tilde{n}_j. \quad (6) \end{aligned}$$

---

[9] Since this model has a structure of a game between the firm and many workers, optimal decisions should be dependent on the past entire history. In this sense, the strategy space is restricted to the stationary policy of Wolinsky (2000) in the present model.

[10] Since we assume stationary policy, the bargained wage only depends on the current state variables. We also note that the wage (w) is also a function of the vacancies of v_j. However, we can ignore this effect in the current bargaining solution. This is because of the additive separability between vacancies and employment, and can be verified by direct calculation.

Under our assumption of unbounded support of η, the solution is interior, thereby leading to endogenous job destruction in all cases (i.e., $\tilde{n}_j < n_j$). In addition, the following first-order conditions are obtained:[11]

$$(v_j) \qquad c = q(\theta) E_{s'|s} J_n(n_j', s')\,\beta, \tag{7}$$

$$(\tilde{n}_j) \qquad af'(\tilde{n}_j) - w^r(\tilde{n}_j, s, 0) - w_n^r(\tilde{n}_j, s, 0)\tilde{n}_j + (1-\phi)\underline{\eta}_j + (1-\delta_w) E_{s'|s} J_n(n_j', s')\,\beta = 0, \tag{8}$$

$$(\underline{\eta}_j) \qquad (1-\phi)(\Psi(\underline{\eta}_j) - \underline{\eta}_j) = \zeta_j. \tag{9}$$

Here, ζ_j is a Lagrangean multiplier for the restriction of $\tilde{n}_j = \bar{G}(\underline{\eta}_j) n_j$, and J_n and $w^r{}_n$ are the partial derivatives of the value function and the wage rate with respect to employment. We can also get the following conditions from the optimality:[12]

$$J_n(n_j, s) = \zeta_j \bar{G}(\underline{\eta}_j) = (1-\phi)\int_{\underline{\eta}_j}^{\infty} [\eta - \underline{\eta}_j]\, dG(\eta). \tag{10}$$

Note that $J_n \geq 0$ from the above expression. Intuitively, if $J_n < 0$, the firm will increase its value immediately by reducing the size of employment.

2.2 Workers

There is one measure of workers in the economy. The value function of the unit measure of unemployed workers is denoted as $U(s)$ and depends on the aggregate state variables:

$$\begin{aligned} U(s) &= b + q(\theta)\theta\Big[E_j^* E_{\eta_i', s'|s} \max\Big(W(n_j', s', \eta_i'), U(s')\Big)\Big]\beta + (1 - q(\theta)\theta) E_{s'|s} U(s')\beta, \\ &= b + q(\theta)\theta\Big[E_j^* E_{\eta_i', s'|s} \max\Big(W(n_j', s', \eta_i') - U(s'), 0\Big)\Big]\beta + E_{s'|s} U(s')\beta. \end{aligned} \tag{11}$$

[11] We use the fact that $\Psi'(\eta) = g(\eta)/\bar{G}(\eta)\left(\Psi(\eta) - \eta\right)$ for rearranging the conditions.

[12] We can obtain this by taking the derivative of (4) with respect to n_j.

Here the operator of E^*_j takes the weighted average:

$$E_j^* x_j = \int_j x_j \frac{v_j}{v^M} \, d j.$$

The flow value of an unemployed worker is the sum of the constant unemployment benefit, b, and the discounted value of the expected benefits in the next period. The unemployed worker finds a job with probability $q(\theta)\theta$. When workers are employed, their value will be $W(n',s',\eta')$, which depends on the characteristics of the matched firms (n',s') as well as those of the worker (η'). The operator of E^*_j is necessary, since the unemployed worker does not know the future employer and is more likely to be matched with the firm posting more vacancies.

The value of being employed in firm j is defined as W and described in the following way under the stationary policy:

$$W(n_j, \mathrm{s}, \eta_i) \;=\; w^r(n_j, \mathrm{s}, \eta_i) + (1-\delta_w) E_{s',\eta_i'|s} \max\Big[W(n_j', \mathrm{s}', \eta_i'), U(\mathrm{s}')\Big]\beta + \delta_w E_{s'|s} U(\mathrm{s}')\,\beta. \quad (12)$$

The value of employed workers is made up of the flow income of w^r and the value in the next period. They are unemployed either when η'_i is below the critical level of firing or when the exogenous shock arrives with probability δ_w in the next period.

2.3 Negotiation

After the size of vacancies (v_j) and the optimal level of job destruction $(n_j\text{-}\tilde{n}_j)$ are determined, $\tilde{n}_j$ surviving workers and the firm bargain over wages. We assume that negotiation starts after job destruction is completed but before production starts.[13] In this model, since each worker has a weight of Δ, the total measure of $\tilde{n}_j$ workers corresponds to N workers where $N=\tilde{n}_j/\Delta$.[14] We also define the set of the employed workers by $\mathbf{C}_w=\{1,2,3,\ldots,N\}$.

We employ the bargaining solution developed by Stole and Zwiebel (1996a) where a firm bargains with N workers. This is a simple extension

[13] This is the natural timing of negotiation since renegotiation can occur at any time before production starts in the model and binding contracts are not available.

[14] Clearly, we ignore the integer problem by assuming that Δ is chosen appropriately.

of the Nash bargaining solution to the multiple worker case. The firm bargains with the workers individually. The firm starts bargaining with worker 1. If they reach agreement, then the firm negotiates with worker 2. This process continues until worker N as long as they reach agreement. However, if bargaining fails with worker i, then this worker leaves the firm, and the remaining N-1 workers and the firm start bargaining again. In this case, the firm needs to renegotiate with worker 1. When all of the workers and the firm reach agreement, the wages are paid according to the agreement.

For each bargaining stage, we use the following Nash bargaining solution between the firm and worker i:

Assumption 1 (Bargaining solution)

$$\phi\Big(\Delta_i J(n, \mathrm{s}, \eta_i)\Big) = (1-\phi)\Big(W(n, \mathrm{s}, \eta_i) - U(\mathrm{s})\Big)\Delta \quad \textit{for all } n \le \tilde{n}_j. \tag{13}$$

Here, ϕ is the bargaining power of the worker.

In the Nash bargaining solution, the threat point of this worker is $U(\mathbf{s})$ Δ, because he/she will be unemployed when bargaining fails to reach agreement. Thus, the net increase in the present value from employment is $(W(n,\mathbf{s},\eta_i)\text{-}U(\mathbf{s}))\Delta$. On the other hand, the firm loses Δ labor inputs whose quality is η_i Δ, when the worker leaves the firm. The net change in the firm's present value is denoted by $\Delta_i J(n,\mathbf{s},\eta_i)$. This assumption requires that the total gain from the match is proportionately split between the firm and the worker. Although the foundation of the strategic bargaining game is possible,[15] we follow Wolinsky (2000) and assume the surplus split rule as an axiom.

Note that this condition must hold for all n that are less than $\tilde{n}_j$. This is because, when the firm and worker i fail to reach agreement, the number of the remaining workers becomes N-1 whose measure is $\tilde{n}_j$-Δ. On this off-the-equilibrium path, the firm and the remaining N-1 workers bargain over wages. The bargaining outcome of this stage also depends on the threat point of firms, which is $J(\tilde{n}_j\text{-}2\Delta,\ \mathbf{s})$, because the number of the remaining workers is N-2 when bargaining fails. This process

[15] A few examples are shown in Stole and Zwiebel (1996a) and Wolinsky (1996).

continues to the zero employment level. In this sense, we need to calculate $J(n,\mathrm{s})$ for all $n \le \tilde{n}_j$ and the bargaining solution of (13) must hold for all possible values of n that are less than $\tilde{n}_j$.

The gain from bargaining for the firm, $\Delta_i J(n,\mathrm{s},\eta_i)$, is obtained from the definition of J in (4) by changing the measure of workers of quality η_i by Δ, but keeping the other variables of $\underline{\eta}_j$, v_j, and n constant. [16] After substituting equation (6) into this condition, we have:

$$\begin{aligned}
\Delta_i J(n,\mathrm{s},\eta_i) &= af(n) - af(n-\Delta) + \eta_i\Delta \\
&\quad -\Big(w^r(n,\mathrm{s},0)\,n - w^r(n-\Delta,\mathrm{s},0)\,(n-\Delta) + \phi\eta_i\Delta\Big) \\
&\quad +E_{\mathrm{s}'|\mathrm{s}}\Big[J\Big(q(\theta)v_j + (1-\delta_w)n,\mathrm{s}'\Big) - J\Big(q(\theta)v_j + (1-\delta_w)(n-\Delta),\mathrm{s}'\Big)\Big]\,\beta. \\
&\doteq af'(n)\Delta - \Big(w^r(n,\mathrm{s},0) + w^r_n(n,\mathrm{s},0)n\Big)\Delta + (1-\phi)\eta_i\Delta \\
&\quad +(1-\delta_w)\,E_{\mathrm{s}'|\mathrm{s}}\,J_n(n',\mathrm{s}')\,\beta\Delta. \qquad (14)
\end{aligned}$$

Here, $n' = q(\theta)\ v_j + (1\text{-}\delta_w)\ n$. The equality holds when we take the limit of $\Delta \to 0$.

In order to derive a tractable bargaining solution, Assumption 1 is important, but it is not sufficient in the dynamic context. The following property is crucial.

Lemma 1 *For all* (n, θ, v_j) *and* s', *we have* $J_n(n',\mathrm{s}')\Delta = E_{\eta_i'}\Delta_i\ J(n',\mathrm{s}',\eta_i')$, *where* $n'=q(\theta)v_j+(1\text{-}\delta_w)n$.

Proof. See the Appendix.

In general, the marginal gain from one worker ($\Delta_i J(n,\mathrm{s},\eta_i)$) is different from the marginal benefit of employing one worker, $J_n(n,\mathrm{s})$. This lemma, however, shows that these two coincide in the next period, thereby leading to the simplified bargaining solution.[17]

[16] In the negotiation stage, firms already determine these variables.

[17] Note that this relation does not hold in the current period. In the current period, some workers already leave the firm after some rounds of negotiation off the equilibrium path, and the remaining number of workers is fewer than the number initially employed. Since the firm determines the posted number of vacancies (v_j) and the critical level of job destruction ($\underline{\eta}_j$) before the negotiation starts, the envelope theorem fails to hold in this out-of-equilibrium path in the current period. But because the workers and the firm behave optimally in the next period and thereafter, even in the off-the-equilibrium path, we can apply the envelope theorem for the

In order to derive the explicit form of the bargaining solution, we first show that the value of being unemployed is rearranged as follows from (7), (11), and (13):

$$U(\mathrm{s}) - E_{\mathrm{s}'|\mathrm{s}}\, U(\mathrm{s}')\, \beta = b + c\frac{\phi}{1-\phi}\theta. \tag{15}$$

We compare the two equations of (12) and (14) by substituting the above expression:

$$\begin{aligned}
(1-\phi)\Big(W(n,\mathrm{s},\eta_i) - U(\mathrm{s})\Big)\Delta \;&=\; (1-\phi)\Big(w^r(n,\mathrm{s},0) - b - c\frac{\phi}{1-\phi}\theta\Big)\Delta + \phi(1-\phi)\eta_i\Delta \\
&\quad +(1-\phi)(1-\delta_w)E_{\mathrm{s}',\eta_i'|\mathrm{s}} \max\Big[W(n',\mathrm{s}',\eta_i') - U(\mathrm{s}'),0\Big]\beta\Delta, \\
\phi\Delta_i\, J(n,\mathrm{s},\eta_i) \;&\doteq\; \phi\Big(af'(n) - w^r(n,\mathrm{s},0) - w_n^r(n,\mathrm{s},0)n\Big)\Delta \\
&\quad +\phi(1-\phi)\eta_i\Delta + \phi(1-\delta_w)\, E_{\mathrm{s}'|\mathrm{s}}\, J_n(n',\mathrm{s}')\,\beta\Delta.
\end{aligned}$$

The left-hand sides of the above two equations are the same by Assumption 1, and the last terms on the right-hand sides are identical by Lemma 1:

$$\phi J_n(n',\mathrm{s}')\Delta = \phi E_{\eta_i'}\Delta_i\, J(n',\mathrm{s}',\eta_i') = (1-\phi)E_{\eta_i'} \max\Big[W(n',\mathrm{s}',\eta_i') - U(\mathrm{s}'),0\Big]\Delta.$$

After subtracting the two equations, we have the following differential equation by taking the limit of $\Delta \to 0$:

$$\phi\Big[af'(n) - w^r(n,\mathrm{s},0) - n\, w_n^r(n,\mathrm{s},0)\Big] = (1-\phi)\Big[w^r(n,\mathrm{s},0) - b - c\frac{\phi}{1-\phi}\theta\Big], \quad \text{for all } n \le \tilde{n}_j.$$

Although the model includes heterogeneous workers and several control variables, the bargaining solution still retains the desirable property in Stole and Zwiebel (1996a), Smith (1999) and Cahuc and Wasmer (2001) in the sense that the future value functions do not affect the current wage level.

By imposing the terminal condition,[18] this differential equation has

---

next period and thereafter.

[18] The firm pays no wage payment when there is no worker. This is expressed as $\lim_{v\to 0} w^r(v,\mathrm{s},0)\, v = 0$. That implies $\lim_{v\to 0} w^r(v,\mathrm{s},0)\, v^{1/\phi} = 0$ for $0<\phi<1$.

the following solution:

$$w^r(n,\mathbf{s},0)n^{\frac{1}{\phi}} = \int_0^n af'(\nu)\nu^{\frac{1}{\phi}-1}\,d\nu + (1-\phi)\Big(b + c\frac{\phi}{1-\phi}\theta\Big)n^{\frac{1}{\phi}}. \quad (16)$$

Although this formula is standard in the literature, we show that the solution is robust under general conditions with heterogeneous workers.

The above wage equation is especially simple when we assume the constant elasticity production function of f=a n^{α}.

$$\begin{aligned} w^r(n,\mathbf{s},0) &= \frac{\alpha}{\alpha+1/\phi-1}an^{\alpha-1} + (1-\phi)\Big(b + c\frac{\phi}{1-\phi}\theta\Big), \\ &= \frac{1}{\alpha+1/\phi-1}af'(n) + (1-\phi)\Big(b + c\frac{\phi}{1-\phi}\theta\Big), \\ w^r(n,\mathbf{s},\eta) &= \phi\frac{1}{1-(1-\alpha)\phi}af'(n) + (1-\phi)\Big(b + c\frac{\phi}{1-\phi}\theta\Big) + \phi\eta \quad (17) \end{aligned}$$

Once we know the wage rate, the value functions can be derived immediately. Although it is not necessary, we assume a symmetric equilibrium in the following analysis. By rearranging the first-order conditions of the firms, (7) - (10), with the solution of w, (16), we have the set of the conditions for the symmetric equilibrium:

$$c = (1-\phi)q(\theta)E\int_{\underline{\eta}'}^{\infty}\left(\eta' - \underline{\eta}'\right)G(d\eta')\,\beta, \quad (18)$$

$$(1-\phi)\Big(\frac{\tilde{n}^{-\frac{1}{\phi}}}{\phi}\int_0^{\tilde{n}} af'(\nu)\nu^{\frac{1-\phi}{\phi}}\,d\nu - b + \underline{\eta}\Big) + \frac{c(1-\delta_w-\phi\theta q(\theta))}{q(\theta)} = 0, \quad (19)$$

where $\underline{\eta}$, n', and $\tilde{n}$ follow the constraints:

$$\tilde{n} = \bar{G}(\underline{\eta})n, \quad n' = q(\theta)v + (1-\delta_w)\tilde{n}, \quad \theta = \frac{v}{1-\tilde{n}}. \quad (20)$$

Definition 1 (Symmetric equilibrium) *A symmetric equilibrium is an allocation of* $(n_{t+1},\tilde{n}_t,\underline{\eta}_t,\theta_t, v_t)$ *for* t=0,1,2,... *that satisfies (18), (19), and (20), given the initial value of* n_0.

3 Properties of the Equilibrium

The basic properties in the model are the same as in Stole and Zwiebel (1996a), Smith (1999) and Cahuc and Wasmer (2001). First, this bargaining outcome preserves the important feature of search equilibrium, in the sense that the surplus is shared between the workers and firms. Second, the wage rate coincides with the standard result of the Nash bargaining solution, when the marginal return is constant.[19] However, as equation (17) indicates, the worker gets more than ϕ of the marginal product when $\alpha<1$ (the case of decreasing marginal return). This is because of the interactions in productivity between workers, and the intuition is the same as in Stole and Zwiebel (1996a): when one worker leaves the firm, the remaining workers have more bargaining power because marginal productivity increases. Thus, employment of this worker contributes to production, as well as to reducing the bargaining power of the remaining workers. This additional benefit is shared between the worker and the firm and is why the shared fraction is greater than ϕ.

Third, this bargaining outcome achieves the Shapley value when $\phi=0.5$. This is an extension of Stole and Zwiebel (1996a) to a model with heterogeneous workers. In order to show this equivalence, we consider the coalitional game of one firm and workers whose size is $\tilde{n}_j$. As assumed before, this corresponds to $N=\tilde{n}_j/\Delta$ workers, each of whom has a weight of Δ. They bargain over the total present value of surplus, assuming that the workers and the firm choose the optimal strategies in the future. Denote the set of workers by $\mathbf{C}_w=\{1,2,\ldots,N\}$, the value of worker i by $W^S{}_i(\mathbf{c}_w,\mathbf{s},\eta_i)\ \Delta$, and the value of the firm by $J^S(\mathbf{c}_w, \mathbf{s})$, where $\mathbf{c}_w \subset \mathbf{C}_w$. It is well known that the Shapley value is characterized by the balanced contributions property (Osborne and Rubinstein, 1994, p. 291). In the current model, this property is defined as follows.

Definition 2 (Balanced contributions property) *For any* $\mathbf{c}_w \subset \mathbf{C}_w$ *and* $i, l \in \mathbf{c}_w$,

[19] We can confirm this equivalence by imposing $\alpha=1$ in (17). See Pissarides (2000) for the solution in the standard job search model.

1. $J^S(\mathbf{c}_w, \mathbf{s}) - J^S(\mathbf{c}_w \backslash i, \mathbf{s}) = \big(W_i^S(\mathbf{c}_w, \mathbf{s}, \eta_i) - U(\mathbf{s})\big)\Delta.$

2. $W_i^S(\mathbf{c}_w, \mathbf{s}, \eta_i) - W_i^S(\mathbf{c}_w \backslash l, \mathbf{s}, \eta_i) = W_l^S(\mathbf{c}_w, \mathbf{s}, \eta_l) - W_l^S(\mathbf{c}_w \backslash i, \mathbf{s}, \eta_l).$

It is clear that the value functions of the present model satisfy this condition.

Proposition 1 (Shapley Value) *The value functions of W and $\Delta_i J$ in (12) and (14) satisfy the balanced contributions property when ϕ=0.5. Hence, the wage equation achieves the Shapley value.*

Proof. Let us take any $\mathbf{c}_w \subset \mathbf{C}_w$ and denote its measure by n_c. Then, we define J^S and W^S so that $J^S(\mathbf{c}_w,\mathbf{s})$ - $J^S(\mathbf{c}_w \setminus i,\ \mathbf{s})$ = $\Delta_i J(n_c,\mathbf{s},\eta_i)$ and $W^S{}_i(\mathbf{c}_w,\mathbf{s},\eta_i)=W(n_c,\mathbf{s},\eta_i)$. The first condition of the balanced contributions property is directly satisfied by (13) when ϕ=0.5. The second condition is derived as follows:

$$\begin{aligned} W_i^S(\mathbf{c}_w, \mathbf{s}, \eta_i) - W_i^S(\mathbf{c}_w \backslash l, \mathbf{s}, \eta_i) &= W(n_c, \mathbf{s}, 0) - W(n_c - \Delta, \mathbf{s}, 0), \\ &= W_l^S(\mathbf{c}_w, \mathbf{s}, \eta_l) - W_l^S(\mathbf{c}_w \backslash i, \mathbf{s}, \eta_l). \end{aligned}$$

Q.E.D.

4 Welfare

The efficiency of the equilibrium is investigated in this section. Welfare is affected by two decisions made by the firm: namely, decisions on job creation and decisions on job destruction. In order to isolate the job creation decision, we first consider the model of homogeneous workers and then investigate the case of heterogeneous workers. We focus on the steady states where a and $\mathbf{s}$ are constant.

4.1 Homogeneous workers

When the workers are homogeneous, there is no endogenous job destruction in the steady state, thereby leading to a variant of Smith (1999), although there is no free entry condition in the present model. We show that the allocation is efficient in equilibrium when the bargaining share is chosen appropriately so that the firms choose efficient job creation levels. The modified version of the firm's value function, (4), is described as follows:

$$J(n_j, \mathrm{s}) \;=\; \max_{v_j, n_j'} \; af(n_j) - w(n_j, \mathrm{s}, 0) - cv_j + J(n_j', \mathrm{s}')\,\beta, \tag{21}$$

$$s.t. \qquad n_j' = q(\theta)v_j + (1-\delta_w)n_j, \tag{22}$$

where $w(n_j,\mathrm{s},0)$ is defined in (16). Under the steady state, the first-order condition is described as follows in the symmetric equilibrium:

$$\begin{aligned}\frac{c(r+\delta_w+\phi\theta q(\theta))}{q(\theta)} &= (1-\phi)\Big(\frac{\tilde{n}^{-\frac{1}{\phi}}}{\phi}\int_0^{\tilde{n}} af'(\nu)\nu^{\frac{1-\phi}{\phi}}\,d\nu - b\Big),\\ &= (1-\phi)\Big(af'(\tilde{n}) - b - \tilde{n}^{-\frac{1}{\phi}}\int_0^{\tilde{n}} af''(\nu)\nu^{\frac{1}{\phi}}\,d\nu\Big).\end{aligned} \tag{23}$$

We use integration by parts for the second equality. This optimal condition corresponds to the equilibrium conditions of (19) in the case of heterogeneous workers. Since there is no endogenous job separation in the homogeneous case, we have only one equilibrium condition for job vacancies. This equation, together with the law of motion of employment in the steady state (i.e., $0 = q(\theta)\,\theta\,(1\text{-}n) - \delta\,n$), determines the equilibrium level of (n,θ).

The efficient condition is derived by solving the planner's problem. Under the symmetric assumption that all firms start with the same initial employment level of n, the planner solves the following problem:[20]

$$\max_{v_t, n_{t+1}} \quad \sum_{t=0}^{+\infty}\beta^t\Big[af(n_t) - cv_t + b(1-n_t)\Big], \tag{24}$$

$$\begin{aligned} s.t. \quad & n_{t+1} = q(\theta_t)v_t + (1-\delta_w)n_t,\\ & n_0 \text{ is given.}\end{aligned}$$

The optimal conditions are described as follows in the steady state:

[20] Note that we assume that the planner uses the same employment strategies for all firms, which is optimal, because of the concave technology of my model.

$$\frac{c(r+\delta_w+\epsilon\theta q(\theta))}{q(\theta)} = (1-\epsilon)\Big(af'(\nu)-b\Big), \tag{25}$$

where ϵ is the absolute value of the elasticity of q with respect to θ (i.e., ϵ=-$q'(\theta)\,\theta/q(\theta)$).

Hosios (1990) showed that the equilibrium is efficient when the bargaining power of workers (ϕ) is equal to the elasticity of the matching function (ϵ). After comparing (23) and (25), this is not clearly true in the present model. When ϕ=ϵ, the left-hand side of (23) is smaller when the allocation (n,θ) is efficient because of the concavity of the production function ($f''(n)$<0). Efficiency is, however, achieved in equilibrium by the appropriate choice of ϕ.

Proposition 2 (Homogeneous Workers) *Efficient allocation is achieved in equilibrium with some level of bargaining power (ϕ=ϕ^e), where ϕ^e>ϵ.*

Proof. In equation (23), suppose that n and θ are the efficient levels to satisfy (25). Then, if ϕ=1, the left-hand side of (23) is greater. If ϕ=ϵ, the left-hand side is smaller because of the efficiency condition. By continuity, there is a point to satisfy this equation. By construction, this point is greater than ϵ. *Q.E.D.*

This efficiency result is the same as the job search models that include decreasing marginal return technology (Bertola and Caballero, 1994, for example). The firm has an incentive to overemploy workers in order to reduce the bargaining position of the employed workers. Because of this additional effect in creating job vacancies, the bargaining share of the firms, 1-ϕ, should be set lower than the standard Hosios condition, i.e., 1-ϕ=1-ϵ, in order to reduce the incentive to open job vacancies.

4.2 Heterogeneous workers

When workers are heterogeneous, the firm needs to make two decisions on job creation and destruction. Even in this case, the Hosios condition (ϕ=ϵ) is enough to simultaneously satisfy the two efficiency conditions in job search models with heterogeneous workers (Pissarides, 2000). We show, however, that it is not true in the present model, and the efficiency result always fails to hold.

For this purpose, consider the planner's problem when the workers are heterogeneous. The planner chooses the optimal job destruction ($\underline{\eta}$) as well as the job creation (v):

$$\max_{\underline{\eta}_t, v_t, \tilde{n}_t, n_{t+1}} \quad \sum_{t=0}^{+\infty} \beta^t \Big[af(\tilde{n}_t) + n_t \int_{\underline{\eta}_t}^{\infty} \eta \, dG(\eta) - cv_t + b(1 - \tilde{n}_t) \Big], \tag{26}$$

$$s.t. \quad n_{t+1} = q(\theta_t) v_t + (1 - \delta_w) \tilde{n}_t, \quad \tilde{n}_t = \bar{G}(\underline{\eta}_t) n_t,$$
$$n_0 \text{ is given.}$$

The optimal conditions are described as follows in the steady state:

$$c = (1 - \epsilon) q(\theta) \int_{\underline{\eta}}^{\infty} \left(\eta - \underline{\eta} \right) dG(\eta) \, \beta, \tag{27}$$

$$(1 - \epsilon) \left(af'(\tilde{n}) - b + \underline{\eta} \right) + \frac{c(1 - \delta_w - \epsilon \theta q(\theta))}{q(\theta)} = 0. \tag{28}$$

When we compare these efficiency conditions with those in the equilibrium of (18) and (19), the following proposition is immediate:

Proposition 3 (Heterogeneous Workers) *The efficient allocation is never achieved in equilibrium when* $f''(n) < 0$.

In order to understand this fact, note that the first conditions of both allocations are identical if $\epsilon = \phi$, i.e., when the Hosios condition holds. However, with this bargaining power, the second equations of the two allocations never coincide, because (19) is modified in the following way:

$$\begin{aligned} 0 &= (1-\phi)\Big(\frac{\tilde{n}^{-\frac{1}{\phi}}}{\phi}\int_0^{\tilde{n}} af'(\nu)\nu^{\frac{1-\phi}{\phi}}\,d\nu - b + \underline{\eta}\Big) + \frac{c(1-\delta_w-\phi\theta q(\theta))}{q(\theta)}, \\ &= (1-\phi)\Big(af'(\tilde{n}) - \tilde{n}^{-\frac{1}{\phi}}\int_0^{\tilde{n}} af''(\nu)\nu^{\frac{1}{\phi}}\,d\nu - b + \underline{\eta}\Big) + \frac{c(1-\delta_w-\phi\theta q(\theta))}{q(\theta)}. \end{aligned}$$

Since the firm has an incentive to overemploy workers, smaller bargaining power should be given to firms to reduce the incentives of overemployment. However, the first equation that regulates job separation is violated in this case. The modified Hosios condition is not enough to satisfy the two conditions simultaneously. This efficiency result stands in contrast with standard job search models with heterogeneous workers.

5 Conclusion

We study a general equilibrium search model when a firm employs many heterogeneous workers and decreasing marginal returns exist in the technology. We generalize the bargaining solution in Stole and Zwiebel (1996a), Smith (1999) and Cahuc and Wasmer (2001) to the model with heterogeneous workers. Although the model includes heterogeneous workers, the wage equation is still simple and tractable. This solution retains several desirable properties, including the equivalence to the Shapley value. In addition, the separation decision is natural since the separation is made when the present value of the surplus is negative. In this sense, it is especially useful in the study of endogenous job separation under various environments.

The endogenous job separation provides an additional condition for efficiency. Without job separation, efficient allocation is possible in equilibrium with a modified Hosios condition where the bargaining share is chosen appropriately to achieve the efficient amount of job creation. When heterogeneity is introduced among workers, job separation arises endogenously, and any equilibrium does not achieve efficient allocation. Although efficiency requires both efficient job separation and job creation conditions, these are not satisfied simultaneously in the present model, because of the interactions between workers within a firm. Endogenous job separation provides an additional source of inefficiency.

Whenever we study a situation where heterogeneous workers interact inside the firm, the standard job search model should be modified. This modification is not easy, since we explicitly need to consider the consistent bargaining solution inside the firm. The present model provides a tractable framework for studying the implications of endogenous job separation.

Appendix

Proof of Lemma 1.

In the bargaining stage, $(v_j, \underline{\eta}_j, \tilde{n}_j)$ is already determined. Suppose that the employment level of the firm is n, where $n \leq \tilde{n}_j$. When $n < \tilde{n}_j$, some workers already leave the firm due to the failure of negotiation. First, we calculate the expected gain from bargaining in the next period, i.e., $E_{\eta'_i} \Delta_i J(n', s', \eta_i')$. The firm and the workers assume that they behave optimally in the next period, in the sense that the firm will make the optimal choices of $(v_j', \underline{\eta}_j', \tilde{n}_j')$, given the state of (n', s', η_i'). In addition, they assume that there will be no separation during the bargaining stage when $\eta'_j \geq \underline{\eta}'_j$ in the next period:

$$
\begin{aligned}
E_{\eta'_i}\Delta_i J(n', s', \eta'_i) &= \int_{\underline{\eta}'_j}^{\infty} \Big[af'(\tilde{n}'_j)\Delta - \Big(w^r(\tilde{n}'_j, s', 0) - w^r_n(\tilde{n}'_j, s', 0)\tilde{n}'_j + \phi\eta'_i\Big)\Delta \\
&\qquad + \eta'_i\Delta + (1-\delta_w)\, E_{s''|s'}\, J_n(n'', s'')\,\beta\Delta\Big]\, dG(\eta'_i), \\
&= (1-\phi)\int_{\underline{\eta}'_j}^{\infty} \Big[\eta'_i - \underline{\eta}'_j\Big]\, dG(\eta'_i)\Delta.
\end{aligned}
$$

In order to obtain the second equation, we use the fact that the firm makes the optimal decision in the next period. We can thus apply the first-order conditions of (8) in the next period. Therefore, $E_{\eta'_i} \Delta_i J(n', s', \eta_i')$ $= J_n(n', s')\ \Delta$ by (10). *Q.E.D.*

References

ACEMOGLU, D. (2001): "Good Jobs versus Bad Jobs," *Journal of Labor Economics*, 19(1), 1–21.

ANDOLFATTO, D. (1996): "Business Cycles and Labor-Market Search," *American Economic Review*, 86(1), 112–132.

BERTOLA, G., AND R. J. CABALLERO (1994): "Cross-Sectional Efficiency and Labour Hoarding in a Matching Model of Unemployment," *Review of Economic Studies*, 61(3), 435–456.

BERTOLA, G., AND P. GARIBALDI (2001): "Wages and the Size of Firms in Dynamic Matching Models," *Review of Economic Dynamics*, 4(2), 335–368.

BLANCHARD, O. J., AND N. KIYOTAKI (1987): "Monopolistic Competition and the Effects of Aggregate Demand," *American Economic Review*, 77(4), 647–660.

CAHUC, P., F. MARQUE, AND E. WASMER (2004): "A Theory of Wages and Labour Demand with Intra-firm Bargaining and Matching Frictions," *C.E.P.R. Discussion Papers*, 4605.

CAHUC, P., AND E. WASMER (2001): "Does Intrafirm Bargaining Matter in the Large Firm's Matching Model?" *Macroeconomic Dynamics*, 5, 742–747.

CHÉRON, A., AND F. LANGOT (2000): "The Phillips and Beveridge Curves Revisited," *Economics Letters*, 69(3), 371–376.

COOLEY, T. F., AND V. QUADRINI (1999): "A Neoclassical Model of the Phillips Curve Relation," *Journal of Monetary Economics*, 44(2), 165–193.

DAVIS, S. J. (2001): "The Quality Distribution of Jobs and the Structure of Wages in Search Equilibrium," *NBER Working Papers*, 8434.

DE FONTENAY, C. C., AND J. S. GANS (2003): "Organizational Design and Technology Choice under Intrafirm Bargaining: Comment," *American Economic Review*, 93(1), 448–455.

DEN HAAN, W. J., G. RAMEY, AND J. WATSON (2000): "Job Destruction and Propagation of Shocks," *American Economic Review*, 90(3), 482–498.

HOSIOS, A. J. (1990): "On the Efficiency of Matching and Related Models of Search and Unemployment," *Review of Economic Studies*, 57(2), 297–298.

KOENIGER, W., AND J. PRAT (2007): "Employment Protection, Product Market Regulation and Firm Selection," *Economic Journal*, 117(521), F302–332.

KRAUSE, M., AND T. A. LUBIK (2007): "Does Intra-Firm Bargaining Matter for Business Cycle Dynamics?" *Deutsche Bundesbank, Discussion Paper Series 1: Economic Studies*, 17.

KUDOH, N., AND M. SASAKI (2007): "Employment and Hours of Work," *COE Discussion Paper Series*, 220.

MARIMON, R., AND F. ZILIBOTTI (2000): "Employment and Distributional Effects of Restricting Working Time," *European Economic Review*, 44(7), 1291–1326.

MERZ, M. (1995): "Search in the Labor Market and the Real Business Cycle," *Journal of Monetary Economics*, 36(2), 269–300.

MORTENSEN, D., AND C. PISSARIDES (1994): "Job Creation and Job Destruction in the Theory of Unemployment," *Review of Economic Studies*, 61(3), 397–415.

OSBORNE, M. J., AND A. RUBINSTEIN (1994): *A Course in Game Theory*. The MIT Press, Cambridge, MA.

PISSARIDES, C. A. (2000): *Equilibrium Unemployment Theory*. The MIT Press, Cambridge, MA, 2nd edn.

ROTEMBERG, J. J. (2006): "Cyclical Wages in a Search-and-Bargaining Model with Large Firms," *NBER Working Papers*, 12415.

SMITH, E. (1999): "Search, Concave Production, and Optimal Firm Size," *Review of Economic Dynamics*, 2(2), 456–471.

STOLE, L. A., AND J. ZWIEBEL (1996a): "Intra-Firm Bargaining under Non-Binding Contracts," *The Review of Economic Studies*, 63(3), 375–410.

—— (1996b): "Organizational Design and Technology Choice under Intrafirm Bargaining," *American Economic Review*, 86(1), 195–222.

TRIGARI, A. (2004): "Equilibrium Unemployment, Job Flows and Inflation Dynamics," *European Central Bank Working Paper Series*, 304.

WALSH, C. (2003): "Labor Market Search and Monetary Shocks," in *Elements of Dynamic Macroeconomic Analysis*, ed. by S. Altug, J. Chadha, and C. Nolan. Cambridge University Press, Cambridge, UK.

WOLINSKY, A. (1996): "A Theory of the Firm with Non-Binding Employment Contracts," *Northwestern University, Center for Mathematical Studies in Economics and Management Science, Discussion Paper*, 1166.

—— (2000): "A Theory of the Firm with Non-Binding Employment Contracts," *Econometrica*, 68(4), 875–910.