Comparison between experimental and theoretical visualization of thermal convection using the photoelastic effect

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Abstract

Under the influence of gravity, thermal convection is created in a rectangular container that is cooled at the top and heated at the bottom. Instead of visualization by mixing tracers that may affect convection cells, we introduce a visualization technique using the photoelastic effect of thermal stress in the acrylic side walls of the container.

The steady convective circulation developing in the flow produced by heating the bottom and cooling the top of a fluid layer is known as a Bénard cell(1). Either shadowgraph or Schlieren photography has been used as a method to visualize cells without mixing tracers. However, they require an extensive set of experimental apparatus, so we introduce here another simpler method to visualize Bénard cells without tracers(2), which is based on the photoelastic effect reflecting thermal stresses. Suppose that the acrylic side walls of the container consist of an isotropic material satisfying Duhamel-Neumann's relation(3),

$$\sigma_{i,j} = \left(\lambda\epsilon_{k,k} - 3\alpha KT\right)\delta_{i,j} + 2\mu\epsilon_{i,j} ,$$

where σ is the stress tensor, ϵ is the strain tensor, λ, μ are the Lame's constants, α is the coefficient of linear thermal expansion, K is the bulk modulus, and T is the temperature field.

As shown in Fig. 1(a), a convection cell is sandwiched between two polarized plates (polarizer and analyzer). A shadow pattern of cells under the photoelastic effect of the side wall is observed from the side opposite to the light source. First, a light ray emitted from the light source is polarized into a vertical component by the polarizer. When the ray passes through an acrylic wall of the container, which is subject to thermal stress under the influence of the temperature field of the cell, the ray undergoes birefringence in the acrylic side wall. If the degree of birefringence is large enough, the ray can pass through the analyzer with horizontal polarization. A pair of contrasts in light intensity observed through the analyzer corresponds to a thermal convection cell. In fact, the number of light/dark pairs counted was in relatively good agreement with the number of cells previously counted from visualization of the tracer.

A container (L=400 mm, h=80 mm, w=10 mm) with acrylic side walls is filled with tap water without tracer mixture. A copper plate at the bottom is heated at a constant temperature T_{bottom}, and an aluminum radiator at the top is cooled at T_{top}. The height of the

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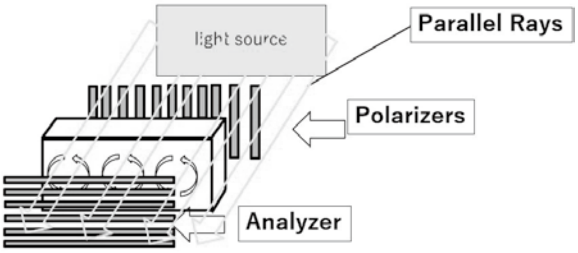
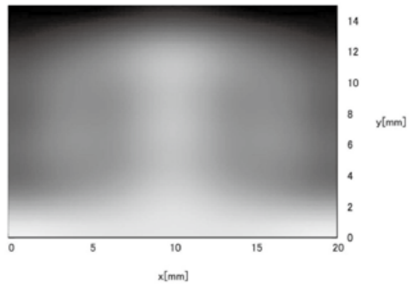


(a) Experimental setup: a convection cell was sandwiched between two polarized plates (polarizer and analyzer).



(b) Temperature field measured by a temperature-sensitive liquid crystal sheet. The temperature of the top and bottom plates was (T_{top}, T_{bottom})=(30°C,15°C).

Fig. 1. Experimental setup and temperature field

fluid layer, h, can be adjusted from 1 to 80 mm by adjusting the height of the radiator. The light source and polarizers are placed behind the container, as shown in Fig. 1(a), and the analyzer is placed in front of the container. Pictures are taken by a camera located in front of the analyzer. The typical height was fixed at $h=25$ mm and 15 mm, and the temperature difference $T_{bottom}-T_{top}$ was 5, 10, 15, and 20℃. The temperature field in the container was measured by a temperature-sensitive liquid crystal sheet via a calibration (Fig. 1(b)).
Suppose that the balance relation of stress on the two-dimensional space may be satisfied in an acrylic side wall. The relation expressed by $\partial\sigma_{i,j}/\partial x_i = 0$ for $j = x, y$ provides

$$\left[(\lambda+2\mu)\frac{\partial^2}{\partial x^2}+\mu\frac{\partial^2}{\partial y^2}\right]u_x+\left[(\lambda+\mu)\frac{\partial^2}{\partial x\partial y}\right]u_y=3\alpha K\frac{\partial T}{\partial x}$$
$$\left[(\lambda+\mu)\frac{\partial^2}{\partial x\partial y}\right]u_x+\left[\mu\frac{\partial^2}{\partial x^2}+(\lambda+2\mu)\frac{\partial^2}{\partial y^2}\right]u_y=3\alpha K\frac{\partial T}{\partial y}$$

where u_x, u_y are components of displacement. We take $T_{0,0}$ = 25°C as the reference temperature under the no pre-strain state. The temperature field may be expanded into Fourier series with coefficients T_{n_x,n_y} as

$$T=\sum\nolimits_{n_x,n_y}T_{n_x,n_y}\cos\left(\frac{2\pi n_x}{L_x}x\right)\cos\left(\frac{2\pi n_y}{2h}y\right)$$

where L_x is the horizontal width of a convection cell (see Fig. 1(b)). The displacement field is also described by Fourier expansion as

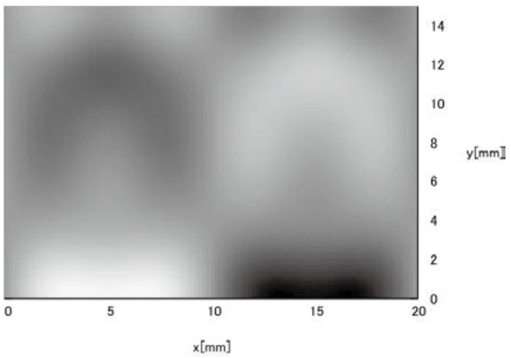
$$(u_x,u_y)=\left(\sum_{n_x,n_y}U_x(n_x,n_y)sin(k_xn_xx)\cos(k_yn_yy),\sum_{n_x,n_y}U_y(n_x,n_y)cos(k_xn_xx)\sin(k_yn_yy),\right)$$

where the wavenumbers are given by $(k_x, k_y) = (2\pi/L_x, \pi/h)$. The coefficients of displacement, U_x and U_y, are determined from T_{n_x,n_y} by the above relation, so that the stress field $\sigma_{i,j}(x, y)$ may be calculated from the displacement field.

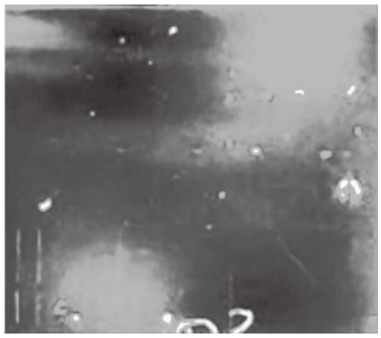
The principal stress is solved by

$$n_\pm=\frac{(\sigma_{x,x}+\sigma_{y,y})\pm\sqrt{(\sigma_{x,x}+\sigma_{y,y})^2+4\sigma_{x,y}^2}}{2}$$

A light ray passing through a photoelastic material is decomposed into electromagnetic wave components along two principal stress directions, each of which experiences a different refractive index due to birefringence. This difference in refractive index causes a relative phase delay between the two components. The amount of light passing through the analyzer is determined by the x component birefringed by the difference in principal stress[3]. The distribution of the intensity of light passing through the analyzer is expressed as $I = \sin^2((n_+ - n_-)\frac{\pi}{\lambda}\Delta z)\ \sin^2 2\theta$, where λ is the wavelength, Δz is the width of the side walls of the container, and θ is the degree of the principal eigenvector against the x axis. The results of light intensity numerically predicted from the temperature field are shown in Fig. 2(a).



(a) Pattern of brightness predicted by numerical calculation.



(b) Pattern of brightness realized experimentally.

Fig. 2. Result of the photoelastic effect

The experimental results are shown in Fig. 2(b). Comparison between the theoretical and experimental results reveals a qualitative similarity, although the theoretical result shows some shadow areas were not evident in the experimental results. We believe that this discrepancy is due to errors in the temperature field. Using the visualization technique of the photoelastic effect, we confirmed that Bénard convection was taking place in a simpler manner than with conventional research equipment.

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