WIDTH OF A DOMAIN WALL USING A QUADRATIC APPROXIMATION OF MAGNETIC MOMENTS

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Abstract:

We considered a quadratic approximation of magnetic moments to calculate the width of a domain wall (DW) of a perpendicularly magnetized nanowire. The calculated DW was compared with the DW estimated by an exact solution for an infinite-length nanowire and a linearly approximated DW. The DW width for the quadratic approximation was approximately equal to the square root of twice the DW width for the linear approximation. The DW energy for the quadratic approximation was less than that for the linear approximation and nearly equal to that for the exact solution.

1 Introduction

A domain wall (DW) is a boundary between magnetic domains in a ferromagnetic (FM) material.[1-3] A straight DW is produced in a perpendicularly magnetized nanowire (inset of Fig. 1).[4] The straight DW moves due to the spin-transfer torque via a spin-polarized current. This applies to the magnetic memory of a DW motion type, such as racetrack memory.[5] The shorter the straight DW width, the larger the memory capacity.

The straight DW magnetization produced in the infinite-length nanowire is calculated using magnetic energies.[6,7] The calculated magnetic moments of the DW are $\theta(x) = 2\arctan(\exp(-x\sqrt{K/A}))$, as shown by the dashed curve in Fig. 1, where $\theta(x)$ is defined as the angle between the easy axis of magnetization and the magnetic moment at position x. K is the effective perpendicular magnetic anisotropy and A is the exchange stiffness constant. Here, the width of the DW is infinite. However, because two or more straight DWs are produced in the nanowire for racetrack memory, these DWs are of finite width. A straight DW produced in a finite-length nanowire also has a finite width.[3,7]

The finite width of the DW is approximated from $\theta(x)$ by a linear approximation of the center of the DW ($\theta(0)$), as indicated by the dashed line in Fig. 1. Consequently, the approximated width of the DW is $\pi\sqrt{A/K}$.[6,7] This value is often used to analyze the magnetic states as the DW width.[2,3,8-10]

The finite width of the DW is calculated from the energy estimated using the linearly approximated $\theta(x)$,[11] as shown by the solid line in Fig. 1. In this case, the DW width is approximated as $\sqrt{2}\pi\sqrt{A/K}$. This value is often used as the DW width to analyze the magnetic states. In the

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linear approximation, the magnetic moments are discontinuities at the boundary between the DW and the magnetic domain. The location where the magnetic moment is discontinuous is not included in the energy calculation.

The two aforementioned DW widths are frequently used to analyze the magnetization state; however, as mentioned above, some limitations arise, namely, the finite width of the DW estimated by the exact solution and the discontinuities for the linearly approximated DW. Therefore, we use a quadratic approximation of the magnetic moments to calculate the DW width of a perpendicularly magnetized nanowire with finite width and continuity at the boundary between the DW and the domain.

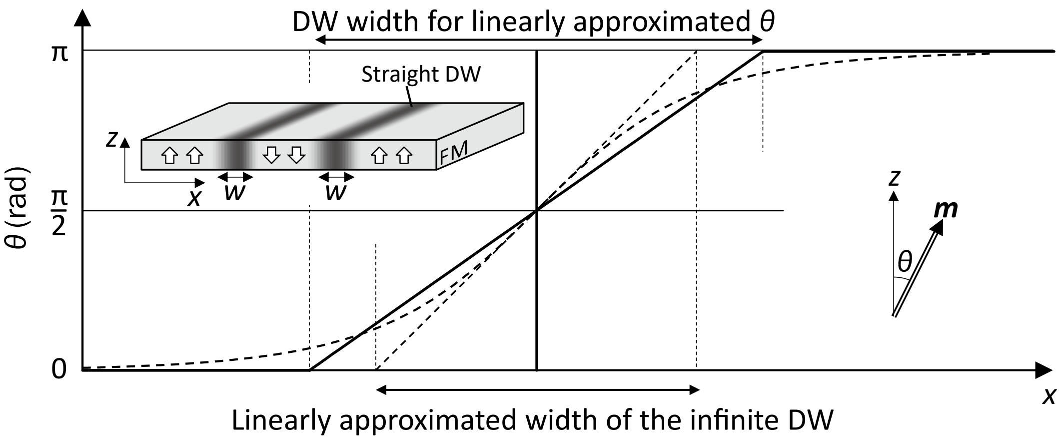

Fig. 1. Schematic illustration of a straight DW produced in a perpendicularly magnetized nanowire and estimated (dashed curve) and linearly approximated θ (solid line) of magnetic moments of DW in a perpendicularly magnetized nanowire.

2 Calculation

We calculate the DW width w approximating $\theta(x)$ to a quadratic equation before the energy calculation. The DW is located at $-\frac{1}{2}w \le x \le \frac{1}{2}w$, and $\theta(x)$ is approximated as follows:

$$\theta(x) = \frac{2\pi}{w^2}(x + \tfrac{1}{2}w)^2 \text{ if } -\tfrac{1}{2}w \le x \le 0;\ \theta(x) = -\frac{2\pi}{w^2}(x - \tfrac{1}{2}w)^2 + \pi \text{ if } 0 \le x \le \tfrac{1}{2}w. \tag{1}$$

Note that $\theta(x)$ is continuous at $x = 0$ and outside DW, as shown in Fig. 2. The DW width is estimated from the exchange energy and the effective anisotropic energy. Under a quasi-one-dimensional model, the linear density of the exchange energy of the DW E_{ex} is given as follows:

$$E_{\mathrm{ex}} \approx \int_{x_0}^{x_1} A\left(\frac{\partial\theta(x)}{\partial x}\right)^2 dx \tag{2}$$

and the linear density of the effective anisotropic energy of the DW E_k is

$$E_{\mathrm{k}} = \int_{x_0}^{x_1} K \sin^2\theta(x)dx, \tag{3}$$

where x_0 and x_1 are the positions of the DW edges.[11,12] Here, $x_0 = -\frac{1}{2}w$ and $x_1 = \frac{1}{2}w$. E_{ex} becomes $\frac{4}{3}\pi^2 A/w$, and E_k becomes $0.313Kw$, where the following approximation is used:

$$\int_0^{\sqrt{\pi/2}} \sin^2\delta^2 d\delta \approx 0.392. \tag{4}$$

Consequently, the linear density of the total energy $E = E_{ex} + E_k$ is given as follows:

$$E = \tfrac{4}{3}\pi^2 A/w + 0.313Kw. \tag{5}$$

Considering $\partial E/\partial w = 0$ of the variation principle, the DW width w is given by

$$w = 2.07\pi\sqrt{A/K} \approx 2\pi\sqrt{A/K}, \tag{6}$$

and the linear density of total energy E is given as follows:

$$E \approx 4.06\sqrt{AK}. \tag{7}$$

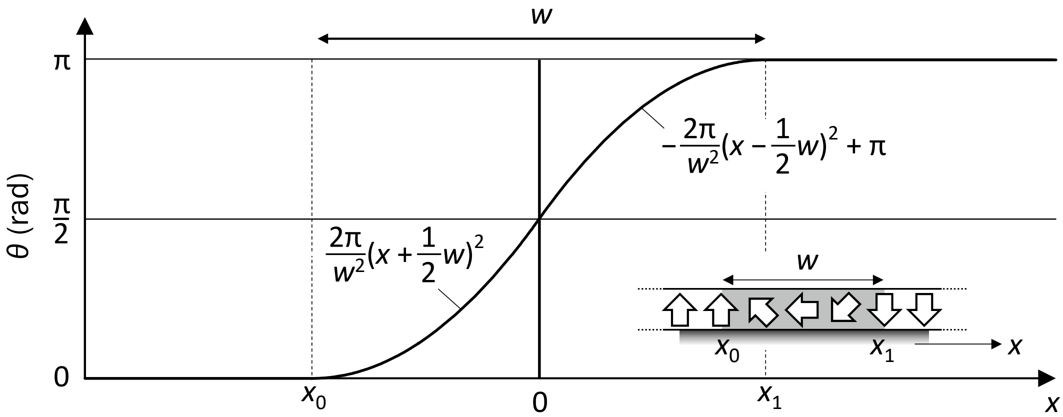

Fig. 2. Quadratically approximated θ of the magnetic moments of DW in the perpendicularly magnetized nanowire.

3 Discussion

The infinite DW has a minimum energy of $4\sqrt{AK}$. The linearly approximated DW has a width of $\sqrt{2}\pi\sqrt{A/K}$ and an energy of $\sqrt{2}\pi\sqrt{AK} \approx 4.44\sqrt{AK}$. The quadratically approximated DW has a width of $2\pi\sqrt{A/K}$ and an energy of $4.06\sqrt{AK}$. The energy of the quadratically approximated DW is smaller than that of the linearly approximated DW and similar to that of the infinite DW. This result indicates that the width of the quadratically approximated DW is better than that of the linearly approximated DW as the approximation of the finite-width DW.

We compared the magnetic states for three DW widths with respect to a micromagnetic simulation based on the Landau-Lifshitz-Gilbert equation.[13] In the simulation, we considered effective magnetic fields comprising an effective perpendicular anisotropy of $K = 0.1$ MJ m^{-3} and the exchange interaction of $A = 1.0$ pJ m^{-1}. The simulations included a nanowire dimension of 100 nm × 10.0 nm × 1.0 nm. A periodic boundary condition was introduced at the xy-plane. Two DWs were produced in the wire.

Figure 3(a) shows the magnetic moments obtained from simulations. The center of the DW was set to $x = 0$, and x was normalized by $\pi\sqrt{A/K}$, which was 10.26 nm. As shown in the figure, the magnetic moments point in the +z-direction ($\theta = 0$) on the left-hand side of the wire. In contrast, the magnetic moments point in the −z-direction ($\theta = \pi$) on the right-hand side of the wire. The magnetic moments at $x \approx 0$ point approximately in the −y-direction. Figure 3(b) shows θ around the DWs. The results of the simulated θ were particularly close to θ for an infinite DW. However, in the simulation, θ becomes 0 and π at $x = -25$ and 25 nm, respectively, because two DWs are produced at $x = 0$ and ±50 nm. The boundary between the domain and the DW was unclear in the simulation and infinite model. The quadratic DW was close to the simulated DW. The DW region can be defined using linear and quadratic approximations.

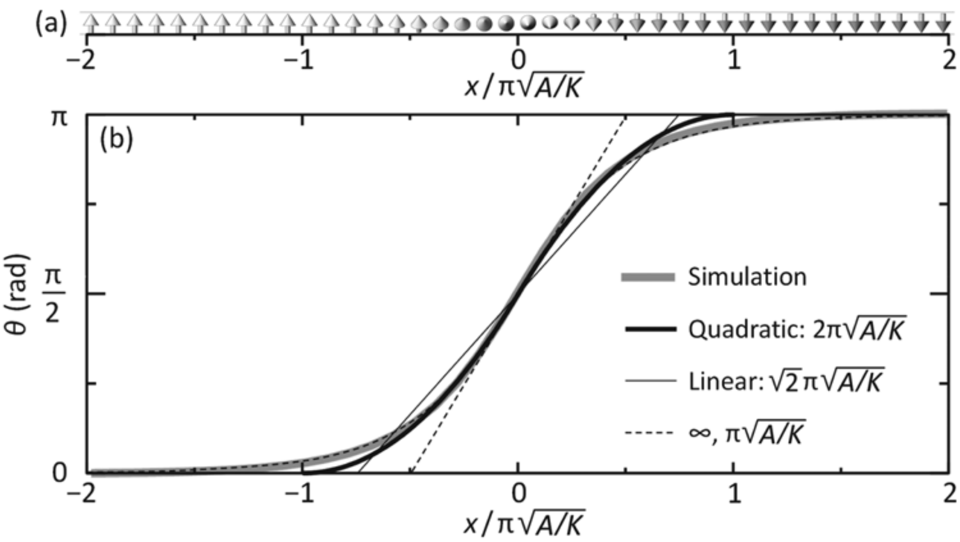

Fig. 3. (a) Excerpt of the magnetic moments around the DW of the simulation result. (b) A comparison of each DW width. Small arrows in (a) represent the direction of magnetic moments.

4 Conclusion

We calculated the width of the quadratically approximated DW produced in the perpendicularly magnetized nanowire, with finite width and continuity at the boundary between the DW and the magnetic domain. The calculated width of the DW was $2\pi\sqrt{A/K}$.

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