

# Effects of Subsidy

Katsuhiko Miyamoto\*

## 1. Introduction

The Japanese Government sometimes subsidizes to enterprises and consumers. The subsidies may give some help on production and consumption. But, these subsidies sometimes disturb economic fairness and economic efficiency.

This paper analyzes the effects of subsidy with mathematical economic model and explains the economic inefficiency of subsidy.

## 2. Economic Model

On this paper, for the sake of simplicity, we assume that the demand and the supply curves are both linear.

$$\text{Demand curve : } p = -ax + b, \quad (1)$$

$$\text{Supply curve : } p = \alpha x + \beta. \quad (2)$$

Here,  $p$ ,  $x$  are the price and the volume of demand (supply) of goods, and  $a$ ,  $b$ ,  $\alpha$ ,  $\beta$  are all positive parameters.

$$a > 0, \quad b > 0, \quad \alpha > 0, \quad \beta > 0, \quad b > \beta.$$

### (A) Perfect Competitive Market

On this paper, we will consider two cases, the one is the perfect competitive market and the other is the monopoly market.

The demand ( $D$ ) and the supply ( $S$ ) curves of the perfect competitive market are shown in figure 1.

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\* Professor of Economics at Kansai University School of Accountancy

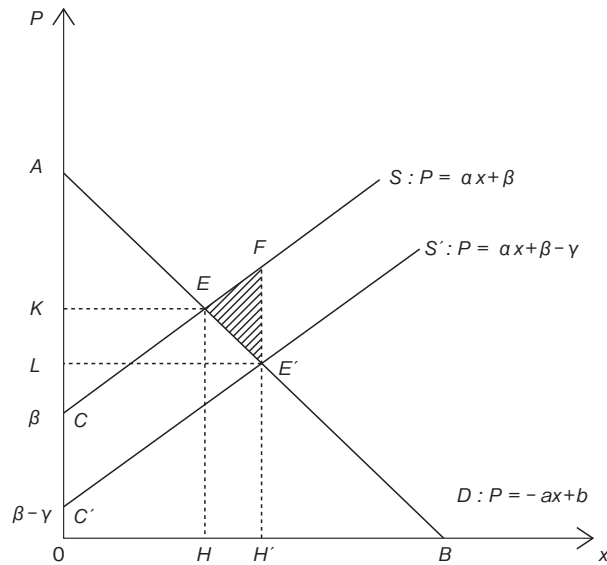


Figure 1

The equilibrium point of the perfect competitive market is the point  $E$ . The equilibrium point is the intersection point of the demand curve and the supply curve. The coordinates of the point  $E$ , that is, the equilibrium point are following,

$$OH = \frac{b - \beta}{a + \alpha} > 0,$$

$$EH = \frac{a\beta + b\alpha}{a + \alpha} > 0.$$

When the government introduces the subsidy, then, the supply curve  $S$  shifts to downward with the unit of subsidy ( $\gamma$ ). The new supply curve after an introduction of the subsidy is  $S'$ .

The equilibrium point after an introduction of the subsidy is the point  $E'$ . The coordinates of the point  $E'$ , that is, the equilibrium point after an introduction of the subsidy, are following,

$$OH' = \frac{b - \beta + \gamma}{a + \alpha},$$

$$E'H' = \frac{a\beta + b\alpha - a\gamma}{a + \alpha}.$$

The coordinates of the point  $F$  and the segment  $HH'$  are following,

$$OH' = \frac{b - \beta + \gamma}{a + \alpha},$$

$$FH' = \frac{a\beta + b\alpha - a\gamma}{a + \alpha},$$

$$HH' = \frac{\gamma}{a + \alpha}.$$

(Welfare Cost ( $WC$ ))

After an introduction of the subsidy, we lose the social surplus. In economics, we call the loss surplus “deadweight loss” or “welfare cost”. Before an introduction of the subsidy, the social surplus is  $\triangle AEC$ , that is, the summation of the consumer’s surplus  $\triangle AEK$  and the producer’s surplus  $\triangle CEK$ . After an introduction of the subsidy, the consumer’s surplus is  $\triangle AE'L$  and the producer’s surplus is  $\triangle E'CL$ , and the subsidy is  $\square E'CCF$ . Therefore, after an introduction of the subsidy, the social surplus  $\triangle AE'C' - \square E'CCF$ . Therefore, the welfare cost, what we call, deadweight loss, is the  $\triangle EE'F$ .

$$\text{Deadweight loss} = \text{Welfare cost } (WC) = \frac{\gamma^2}{2(a + \alpha)}.$$

When the price elasticity of demand ( $a$ ) increases (decreases), the  $WC$  decreases (increases).

$$\frac{\partial WC}{\partial a} = \frac{-2\gamma^2}{\{2(a + \alpha)\}^2} < 0.$$

When the price elasticity of supply ( $\alpha$ ) increases (decreases),  $WC$  decreases (increases).

$$\frac{\partial WC}{\partial \alpha} = \frac{-2\gamma^2}{\{2(a + \alpha)\}^2} < 0.$$

When the subsidy ( $\gamma$ ) increases (decreases),  $WC$  increases (decreases).

$$\frac{\partial WC}{\partial \gamma} = \frac{\gamma}{(a + \alpha)} > 0.$$

(Total Amount of Subsidy ( $TS$ ))

Total amount of subsidy ( $TS$ ) is as follows,

$$\text{Total amount of subsidy } (TS) = \frac{(b - \beta + \gamma) \gamma}{(a + \alpha)}.$$

When the price elasticity of demand ( $a$ ) increases (decreases), the  $TS$  decreases (increases).

$$\frac{\partial TS}{\partial a} = \frac{-(b - \beta + \gamma) \gamma}{(a + \alpha)^2} < 0.$$

When the price elasticity of supply ( $\alpha$ ) increases (decreases),  $TS$  decreases (increases).

$$\frac{\partial TS}{\partial \alpha} = \frac{-(b - \beta + \gamma) \gamma}{(a + \alpha)^2} < 0.$$

When the subsidy ( $\gamma$ ) increases (decreases),  $TS$  increases (decreases).

$$\frac{\partial TS}{\partial \gamma} = \frac{2\gamma}{(a + \alpha)} > 0.$$

When the price elasticity of demand ( $a$ ) is 0,  $WC$  is also 0.

$$WC = \frac{\gamma^2}{2(\infty + \alpha)} \doteq 0.$$

## Effects of Subsidy

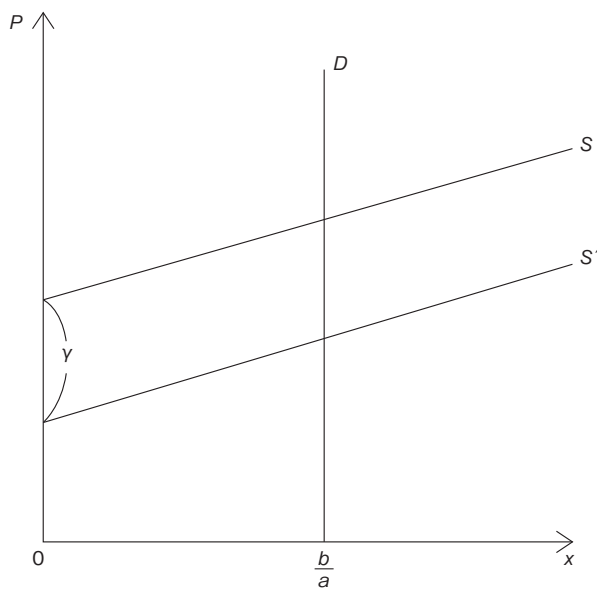


Figure 2

When the price elasticity of demand ( $a$ ) is  $\infty$ , then  $WC$  is as follows,

$$WC = -\frac{\gamma^2}{2a}$$

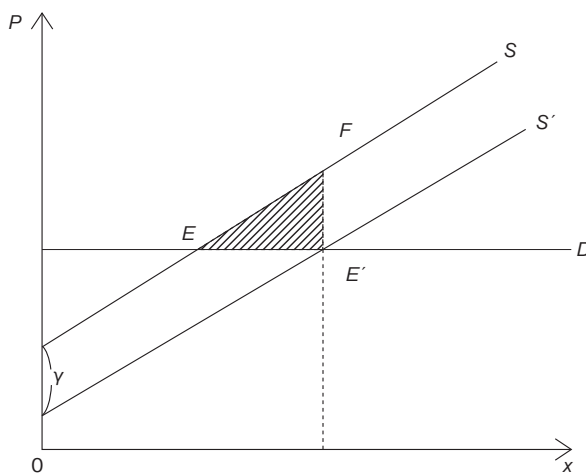


Figure 3

**(B) Monopoly Market**

The monopoly market case is shown on figure 4. The equilibrium condition of the monopoly market is the equality of the marginal cost ( $MC$ ) and the marginal revenue ( $MR$ ). The  $MC$  curve is the supply curve ( $S$ ), and the  $MR$  curve is  $AS$  on figure 4.

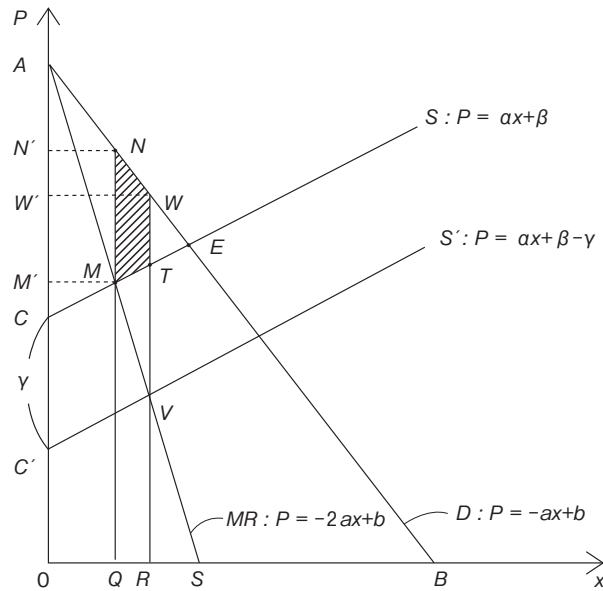


Figure 4

The  $MR$  is as follows,

$$MR : p = \frac{d}{dx} (x \cdot p) = \frac{d}{dx} (-ax + bx) = -2ax + b. \quad (3)$$

The coordinates of the point  $M$ , that is the intersection of the  $MC$  curve and the  $MR$  curve, are following,

$$OQ = \frac{b - \beta}{2a + \alpha} > 0,$$

$$QM' = \frac{2a\beta + b\alpha}{2a + \alpha} > 0.$$

The coordinates of the point  $N$  are as follows,

## Effects of Subsidy

$$OQ = \frac{b - \beta}{2a + \alpha} > 0 ,$$

$$QN = \frac{ab + a\beta + ba}{2a + \alpha} > 0 .$$

After an introduction the subsidy ( $\gamma$ ), the supply curve ( $S$ ) shifts to downward with  $\gamma$ , then the new supply curve ( $S'$ ) is as follows.

$$S' : p = ax + \beta - \gamma .$$

After an introduction the subsidy ( $\gamma$ ), the new intersection of the  $MR$  curve and the new supply curve ( $S'$ ) is point  $V$ . The coordinates of the point  $V$  are as following,

$$OR = \frac{b - \beta + \gamma}{2a + \alpha} > 0 ,$$

$$VR = \frac{2a\beta - 2a\gamma + ba}{2a + \alpha} > 0 .$$

The segment  $RW$  is as follows,

$$RW = \frac{ab + a\beta + ba - a\gamma}{2a + \alpha} > 0 .$$

(Welfare cost ( $WC$ ))

Before an introduction of the subsidy, the social surplus (welfare) is  $\square ANMC$ , and after an introduction of the subsidy, the summation of the social surplus and the subsidy is  $\square AWTC$ .

$$\begin{aligned} \text{New Surplus} &= \triangle AWW' + \square WW'C'V - \square TCC'V \\ &= \square AWTC. \end{aligned}$$

Therefore, the welfare cost ( $WC$ ), that is, deadweight loss, is  $\square NWTM$ .

$$WC = \square AWTC - \square ANMC = \square NWTM$$

$$\square NWTM = \frac{\{2a(b - \beta) - \gamma(a + \alpha)\} \gamma}{2(2a + \alpha)^2} > 0.$$

Therefore, the introduction of subsidy may decrease the social wealth. That is, the introduction of subsidy may bring inefficiency of economy in.

### 3. Conclusions

We got the following conclusions by our economic model and analyses.

In the perfect competitive market, we got the following conclusions.

- (1) The welfare cost ( $WC$ ), that is the deadweight loss, is  $\triangle EE'F$  in figure 1, and an increase of the price elasticity of demand decreases the  $WC$ , and an increase of the price elasticity of supply decreases the  $WC$ .
- (2) When the price elasticity of demand ( $a$ ) increases (decreases), the  $WC$  decreases (increases).
- (3) When the price elasticity of supply ( $\alpha$ ) increases (decreases),  $WC$  decreases (increases).
- (4) When the subsidy ( $\gamma$ ) increases (decreases),  $WC$  increases (decreases).
- (5) When the price elasticity of demand ( $a$ ) increases (decreases), the total amount of subsidy ( $TS$ ) decreases (increases).
- (6) When the price elasticity of supply ( $\alpha$ ) increases (decreases),  $TS$  decreases (increases).
- (7) When the subsidy ( $\gamma$ ) increases (decreases),  $TS$  increases (decreases).

In the monopoly market, we got the following conclusion.

- (8) An introduction of subsidy may decrease the social wealth. That is, the introduction of subsidy may bring inefficiency of economy in.

### References

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