

# Effects of Ad Valorem Tax

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## 1. Introduction

This paper analyzes the effects of an ad valorem tax with a mathematical economic model and figures, and explains the economic inefficiency of an ad valorem tax.

Recently, the Japanese government has been bothering with an increasing budget deficit. The Japanese ratio of an accumulated debt to GDP becomes nearly 190% in 2010. This value is the highest among the other modern countries. Therefore, the Japanese government has to decrease the ratio of an accumulated debt to GDP. Then the Japanese government is planning to increase the rate of the consumption tax in order to increase the government revenue. The consumption tax is one of the general ad valorem taxes. We will study the effects of the ad valorem tax on this paper.

This paper is planning to analyze the effects of an introducing of ad valorem tax. The ad valorem tax is a constant tax rate on the amount of sales.

## 2. Economic Model

In this paper, for the sake of simplicity, we assume that the demand and the supply curves are both linear.

$$\text{Demand curve : } p = -ax + b, \quad (1)$$

$$\text{Supply curve : } p = ax + \beta. \quad (2)$$

Here,  $p$ ,  $x$  are the price and the volume of demand (supply) of goods, and  $a$ ,  $b$ ,  $\alpha$ ,  $\beta$  are all positive parameters.

$$a > 0, \quad b > 0, \quad \alpha > 0, \quad \beta > 0, \quad b > \beta.$$

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### 3. Perfect Competitive Market

#### (1) Before the ad valorem tax

On this paper, we will consider two cases, the one is the perfect competitive market and the other is the monopoly market. First, we will analyze the case of competitive market.

The demand ( $D$ ) and the supply ( $S$ ) curves of the perfect competitive market are shown in figure 1.

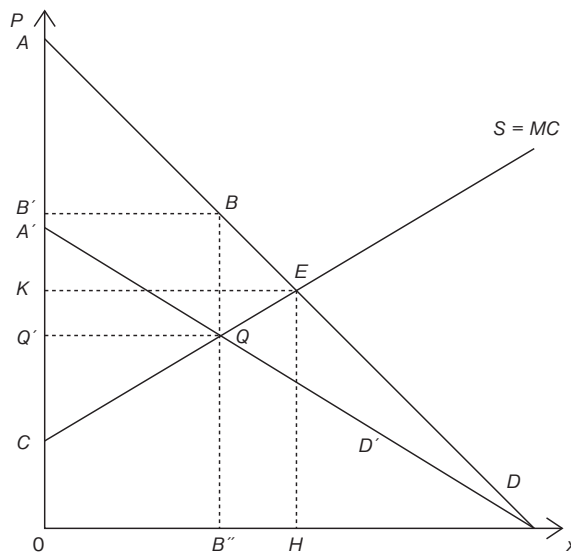


Figure 1

The equilibrium point of the perfect competitive market is the point  $E$ . The equilibrium point is the intersection point of the demand curve and the supply curve. The coordinates of the point  $E$ , that is, the equilibrium point are followings,

$$OH = \frac{b - \beta}{a + \alpha} > 0,$$

$$EH = \frac{a\beta + \alpha b}{a + \alpha} > 0.$$

#### (2) After the ad valorem tax

When the government introduces the ad valorem tax with constant rate  $t$ , or introduces

the consumption tax with constant rate  $t$ , then the demand curve  $D$  shifts to downward with  $\frac{1}{1+t}$ . The new demand curve after an introduction of the consumption tax is  $D'$ .

We assume that the  $D'$  curve is as follows,

$$P = -a'x + b', \quad a' > 0, \quad b' > 0.$$

Then, the  $D$  curve is  $(1+t) \cdot D'$ , that is,

$$P = (1+t) \{-a'x + b'\}. \quad (3)$$

$$\therefore (1+t) a' = a, \quad \text{and} \quad (1+t) b' = b.$$

Therefore,  $a'$  and  $b'$  are as follows.

$$a' = \frac{a}{1+t},$$

$$b' = \frac{b}{1+t}.$$

Then, the new demand curve is as follows,

$$P = \frac{1}{1+t} (-ax + b). \quad (4)$$

This is to say, the new demand curve  $D'$  rises  $\frac{1}{1+t}$  fold of  $D$  curve. That is, the new demand curve  $D'$  shifts to downward from  $D$  curve.

The intersection point  $Q$  of the  $S$  curve and the  $D'$  curve after an introduction of the ad valorem tax is the point  $F$ . The coordinates of the point  $Q$  are following,

$$OR = \frac{b - \beta(1+t)}{a + a(1+t)} > 0,$$

$$QR = \frac{a\beta + b\alpha}{a + a(1+t)} > 0.$$

The coordinates of the point  $B$  are following,

$$OR = \frac{b - \beta (1 + t)}{a + \alpha (1 + t)} > 0,$$

$$BR = \frac{(1 + t) (a\beta + b\alpha)}{a + \alpha (1 + t)} > 0.$$

### (3) Comparisons with the perfect competitive case and the monopoly case

#### (A) Consumer surplus

After an introduction of the ad valorem tax, the consumer surplus is  $\triangle ABB'$ . On the other hand, before an introduction of the ad valorem tax, the consumer surplus is  $\triangle AEK$ . Then, by an introduction of the ad valorem tax, the consumer surplus decreases with  $\square BB'KE$ .

#### (B) Producer surplus

After an introduction of the ad valorem tax, the producer surplus is  $\square BB'CQ$ . On the other hand, before an introduction of the ad valorem tax, the producer surplus is  $\triangle CEK$ . Then, by an introduction of the ad valorem tax, the producer surplus decreases with  $\square EKQ'Q$ .

#### (C) Tax revenue ( $TR$ )

After an introduction of the ad valorem tax, the amount of tax revenue is  $\square BB'Q'Q$ .

$$TR = OB'' \times BQ = \square BB'Q'Q = \frac{t \{(a\beta + \alpha b) (b - \beta (1 + t))\}}{\{a + \alpha (1 + t)\}^2} > 0.$$

#### (D) Social surplus

After an introduction of the ad valorem tax, the social surplus is  $\triangle ABB' + \triangle QQ'C$ . On the other hand, before an introduction of the ad valorem tax, the social surplus is  $\triangle ACE$ .

#### (E) Social surplus and tax revenue

After an introduction of the ad valorem tax, the sum of social surplus and tax revenue is  $\square ABQC$ . On the other hand, before an introduction of the ad valorem tax, the social surplus is  $\triangle ACE$ . Then, the social surplus decreases.

#### (F) Deadweight loss (Welfare Cost ( $WC$ ))

After an introduction of the ad valorem tax, we lose the social surplus. In economics, we call the loss surplus "deadweight loss" or "welfare cost". Before an introduction of the ad valorem tax, the social surplus is  $\triangle AEC$ , that is, the summation of the social surplus  $\triangle AEK$  and the producer's surplus  $\triangle CEK$ . After an introduction of ad valorem tax, the

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consumer's surplus is  $\triangle ABB'$  and the producer's surplus is  $\triangle CQQ'$  and the tax revenue is  $\square BB' CQ$ . Therefore, after an introduction of the ad valorem tax, the social surplus and the tax revenue is  $\square ABQC$ . Therefore, the welfare cost, what we call, deadweight loss is the  $\triangle BEQ$ .

$$\text{Deadweight loss} = \text{Welfare cost (WC)} = \triangle BEQ = \frac{1}{2} \frac{(a\beta + b\alpha)^2 \cdot t^2}{(a + \alpha)\{a + \alpha(1 + t)\}^2}$$

We compare the before case of the ad valorem tax with the after case of the ad valorem tax.

	Before ad valorem tax	After ad valorem tax
Price	$EH$	$BB''$
Amount of sale	$OH$	$OB''$
Consumer surplus	$\triangle AEK$	$\triangle ABB'$
Producer surplus	$\triangle CEK$	$\triangle QQ'C$
Social surplus	$\triangle AEC$	$\triangle ABB' + \triangle QQ'C$
Tax revenue	0	$\square BB' CQ$
Deadweight loss	0	$\triangle BEQ$

(G) Effect of an increase of the ad valorem tax rate

The effect of an increase of the ad valorem tax rate increases the dead weight loss.

$$\frac{\partial \triangle BEQ}{\partial t} = \frac{(a\beta + b\alpha)^2}{a + \alpha} \left\{ \frac{t}{a + \alpha(1 + t)} \right\} \frac{a + \alpha}{\{a + \alpha(1 + t)\}^2} > 0.$$

Thus, an introduction of the ad valorem tax increases the deadweight loss. That is, an introduction of ad valorem brings out the social surplus loss.

## 4. Monopoly

(1) Before the ad valorem tax

In the monopoly market, the equilibrium condition of the monopoly producer is  $MC = MR$ .

We introduce the  $MR$  (Marginal Revenue) curve of the  $D$  curve. That is the  $MR$  line in figure 2.

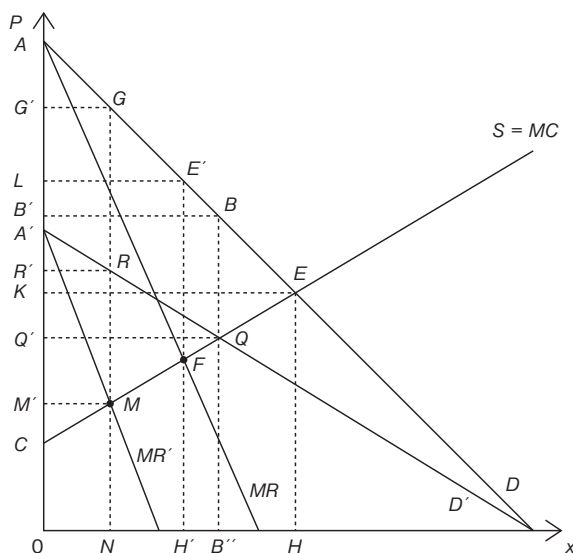


Figure 2

The equilibrium point of monopoly market is the point  $F$ . The equilibrium point is the intersection point of the  $MR$  line and  $MC$  line. The coordinates of the point  $E'$  of the monopoly market is  $(OH', E'H')$ . The price is  $E'H'$  and the amount of sale is  $OH'$ .

- (A) Consumer surplus  
The consumer surplus is  $\triangle AE'L$ .
- (B) Producer surplus  
The producer surplus is  $\square E'LCF$ .
- (C) Social surplus  
The social surplus is  $\square AE'FC$ .

**(2) After the ad valorem tax**

We introduce the  $MR$  (Marginal Revenue) curve of the  $D'$  curve after introducing the ad valorem tax, that line is the  $MR'$  line in figure 2.

The  $MR$  and the  $MR'$  are as follows,

$$R = x \cdot P = x \cdot (-ax + b)$$

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$$MR = \frac{d}{dx} x \cdot (-ax + b) = -2ax + b.$$

$$R' = x \cdot (-a'x + b') = x \cdot \frac{(-ax + b)}{1 + t}$$

$$MR' = \frac{d}{dx} x \cdot (-a'x + b') = \frac{(-2ax + b)}{1 + t}$$

The new  $MR'$  line becomes parallel to the  $MR$  line.

After an introducing of the ad valorem tax, the equilibrium point is  $M$ , and the market price becomes  $GN$  and the amount of sale becomes  $ON$ .

## (A) Consumer surplus

After an introduction of the ad valorem tax, the consumer surplus is  $\triangle AGG'$ . On the other hand, before an introduction of the ad valorem tax, the consumer surplus is  $\triangle AE'L$ . Then, by an introduction of the ad valorem tax, the consumer surplus decreases with  $\square E'GG'L$ .

## (B) Producer surplus

After an introduction of the ad valorem tax, the producer surplus is  $\square RR'CM$ . On the other hand, before an introduction of the ad valorem tax, the producer surplus is  $\square E'FCL$ .

## (C) Tax revenue

After an introduction of the ad valorem tax, the tax revenue ( $TR$ ) is  $\square GG'R'R$ .

## (D) Social surplus

After an introduction of the ad valorem tax, the social surplus is  $\triangle AGG' + \square RR'CM$ . On the other hand, before an introduction of the ad valorem tax, the social surplus is  $\square ACFE'$ .

## (E) Social surplus and tax revenue

After an introduction of the ad valorem tax, the sum of social surplus and tax revenue is  $\square ACMG$ . On the other hand, before an introduction of the ad valorem tax, the social surplus is  $\square ACFE'$ . Then the social surplus decreases with  $\square E'FMG$ .

(F) Deadweight loss (Welfare Cost ( $WC$ ))

After an introduction of the ad valorem tax, we lose the social surplus. In economics, we call the loss surplus "deadweight loss" or "welfare cost". Before an introduction of the

ad valorem tax, the summation of the social surplus  $\square AE'FC$ . After an introduction of ad valorem tax, the consumer's surplus is  $\triangle AGG'$  and the producer's surplus is  $\square RR'CM$  and the tax revenue is  $\square GG'R'R$ . Therefore, after an introduction of the ad valorem tax, the deadweight loss is the  $\square E'FMG$ .

$$\text{Deadweight loss} = \text{Welfare cost (WC)} = \square E'FMG.$$

### (3) Comparisons with the case of before the ad valorem tax and the case of after the ad valorem tax

	Before ad valorem tax	After the ad valorem tax
Price	$E'H'$	$GN$
Amount of sale	$OH'$	$ON$
Consumer surplus	$\triangle AE'L$	$\triangle AGG'$
Producer surplus	$\square E'FCL$	$\square RR'CM$
Social surplus	$\square ACFE$	$\triangle AGG' + \square RR'CM$
Tax revenue	0	$\square GG'R'R$
Deadweight loss	0	$\square E'FMG$

## 5. Conclusions

We got the following conclusions by our economic model and analyses.

### 5-1. Perfect competitive market,

- (1) The government introduces the ad valorem tax, then the equilibrium market price increases from  $EH$  to  $BB''$ , the equilibrium volume of sale decreases from  $OH$  to  $OB''$ , the consumer surplus decreases from  $\triangle AEK$  to  $\triangle ABB'$ , the producer surplus decreases from  $\triangle CEK$  to  $\triangle QQ'C$ , and the deadweight loss ( $\triangle BEQ$ ) comes into existence.
- (2) The government increases the ad valorem tax rate, then the deadweight loss increases.
- (3) Therefore, an introduction of the ad valorem tax increases the deadweight loss. Then, the higher the ad valorem tax rate is, the more the social welfare loss is. That is, an introduction of ad valorem tax brings out the social surplus loss.

### 5-2. Monopoly market

- (1) The government introduces the ad valorem tax in the monopoly market, then the equilibrium market price increases from  $EH'$  to  $GN$ , the equilibrium volume of sale decreases from  $OH'$  to  $ON$ , the consumer surplus decreases from  $\triangle AE'L$  to  $\triangle AGG'$ , the producer



surplus decreases from  $\square E'FCL$  to  $\square RR'CM$ , and the deadweight loss increases with  $\square E'FMG$ .

- (2) Therefore, an introduction of the ad valorem tax in the monopoly market increases the deadweight loss. That is, an introduction of ad valorem tax in the monopoly market decreases the social welfare and brings out the social surplus loss.

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