A PATTERN DYNAMICS APPROACH TO TWO-PHASE FLOW DYNAMICS

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Abstract

Two-phase flow shows, intrinsically, fluctuations in representative parameters such as void fraction and pressure drop. Thus, in the conventional formulations, statistical averaging in time and space is introduced to identify parameters, but the modeling is not suitable for discussion of void fraction fluctuation and flow regime transition. The proposed pattern dynamics approach has been developed in order to break through such problems and to be universally applicable to any flow regime transition. This paper describes the concept behind this new approach to two-phase flow dynamics.

1. Background and Brief History of Two-Phase Flow Dynamics Research

"Two-phase flow" is the flow of a mixture of gas and liquid, an important type of flow present in boilers, nuclear reactors and chemical reactors. Such two-phase flow becomes very important, not only at the design stage, but also during the operation of such plants. Flow stability problems are quite typical in nuclear reactors. In such a case, the transient behavior of two-phase flow is of prime importance in understanding and preventing the incipience of flow instability. This transient behavior has been widely investigated so far under the title of "two-phase flow dynamics".

Two-phase flow dynamics and related topics were first investigated in Japan with reference to the automatic control of the once-through boiler[1] in the 1960s. This was mainly because the once-through boiler has no steam drum, being different from the natural circulation boiler, a major type of boiler at that time. The natural circulation boiler has a large heat capacity, brought about by the stream drum, suited to steady operation; while the once-through boiler is rather sensitive to change - e.g. inlet flow rate, furnace heat release rate or steam pressure. The transient behavior (e.g. of steam pressure) is a complicated function of these parameters[1]. In the 1980s, when such once-through boilers became fully-developed, nuclear power grew in importance and became a major type of power generation. The growing environmental and/or energy problems enhanced innovation in power engineering. This is typically exemplified by the introduction of partial-load operation and daily start-and-stop (DSS) operation of the once-through boilers. Such

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boiler operation essentially introduced variable-pressure for more efficient use of energy - i.e. fossil fuels, including oil and coal. Subsequently, a large-scale unit would be able to be reach full-load operation taking only a few hours. Research on two-phase flow dynamics and related heat transfer contributed a great deal towards the realization of such boiler operation[2,3].

As to nuclear power technologies, a boiling water reactor (BWR) was developed in the 1950s in the U.S. In a BWR, the void reactivity (i.e. a coupling between the void fraction and power generation by nuclear reaction) may suffer from two-phase flow dynamics. Thus much research was conducted - both basic and advanced - mainly in the US. When flow instability is initiated in the reactor core, steady operation is not possible any more due to the large fluctuation of flow, and thus heat generation. Thermal and mechanical fatigue may be expected in the reactor system. A typical example of a research paper related to such flow instability was the work done by Wallis & Heasely[4] which appeared in *ASME Transactions* in 1961. The year 1961 was one year after commission of the first commercial plants using BWR or PWR (pressurized water reactor) in the U.S. In Japan, on the other hand, the first BWR plant began operation in 1970, 10 years after the first power generation plants in the U.S.[3]

The first systematic analysis of the two-phase flow instabilities in BWR was conducted by Hayama[5] in Japan. He published a series of papers from 1962 to 1966. His research covered a wide range of problems, and shed some light on the two-phase flow instability problems within a framework of control theory and mechanical vibration. In 1967, a milestone in the study of two-phase flow dynamics was the "International Symposium on Two-Phase Flow Dynamics", held in Eindhoven, that essentially covered really all of the important areas related to nuclear power and reactor safety.

On viewing the methodologies for two-phase flow instabilities, one-dimensional flow modeling was predominant owing to the fact that most of the discussions focused on channel flow. In such one-dimensional flow modeling, conservation equations of mass and energy were first linearized by applying a small-perturbation method - i.e. considering small deviations of variables from steady state values - and then later Laplace transformed[6-8]. The transformed equations integrated over the channel length gave so-called "transfer functions" between the inlet flow perturbation and output variables (such as velocity and density) in the frequency domain. These transfer functions were then substituted into the linearized momentum conservation equation, so as to give the transfer function of pressure drop against the inlet flow perturbation. This transfer function is used in analyzing the system stability with the help of classical control theory. The dynamic interaction between inlet flow perturbation and pressure drop is well represented by a block diagram of feedback control system, which is easy to analyze using Nyquist or other methods. One of the typical examples is "DYNAM" code[9], which is used in steam generator development for liquid-metal fast breeder reactors (LMFBR), not only in the U.S. and Japan, but also in Germany and France. In the same decade, a numerical simulation code "HYDNA"[10] had been developed in the U.S. This code provided stability analysis in time domain using a finite-difference method. Owing to the limitations of computer capacity at that time, however, most of the stability analyses were conducted less by using such a finite-difference code, and more by using a linearized stability analysis similar to DYNAM code.

In such modelings, system dynamics was formulated analytically based on the distributed-parameter system. This analysis is more suitable for grasping the parameter effect - i.e. scaling - whereas stability analysis must be conducted point by point. This means that even such linearized analysis is inconvenient in understanding the whole image of the two-phase flow dynamics of the system. In reality, even a linearized stability analysis requires from several hundreds to thousand statements, to describe the system dynamics, of the computer program[6-8]. Thus, the simplified stability criterion is far beyond its scope. Probably the first successful criterion was proposed by Ishii & Zuber[11] based on D-partition method applied to the distributed-parameter system dynamics.

Computational methods for analyzing two-phase flow dynamics have since then been highly developed. "TRAC"[12] and "RELAP"[13] codes are typical examples for reactor safety analysis. These codes are well-accepted and are often applied to various types of transients encountered in nuclear reactors. However, these codes include many constitutive relationships or empirical equations. Thus the codes are not suitable for understanding the overall trends of flow instability. Moreover, they may not be correct, in the sense that all the correlations are determined under steady state conditions. The two-phase flow has fluctuating properties even under steady state conditions. Neglecting such instantaneous fluctuations while averaging parameters in time, the conventional modeling, including homogeneous flow, drift-flux and two-fluid models, is constructed so as to be able to define both phases everywhere in time and space domain. Such a concept is referred to as "continuous flow hypothesis" in this paper.

In the 1990s, non-linear dynamics, especially chaotic phenomena, were intensively investigated, and many papers described non-linear aspects of flow instability in boiling channels. In discussing such non-linear phenomena, we essentially used a finite-difference method or alternatives. When the heat flux is uniform along the boiling channel, the enthalpy distribution and specific volume of mixtures, if a homogeneous flow model is applied, become linear functions of the axial coordinate along the channel. In this case, the velocity distribution becomes also a linear function of the axial coordinate along the channel. Then the partial differential equations of the flow field are reduced to ordinary differential equations with non-linearity. They may be easily calculated by means of the Runge-Kutta method[14]. The limit cycle oscillation of the flow instability may, of course, be easily obtained in this analysis, although this modeling is still based on the continuous flow hypothesis.

In the above, discussion was carried out without specifying the type of phenomenon among the flow instabilities, and general, some time historical, aspects were focused on. In the case of "density wave oscillation" in the following discussion, the key parameter in describing two-phase flow dynamics is specified.

2. Void Wave - a Key Factor

Density wave oscillation is the most often-encountered flow instability in boiling channel systems, being induced by the dynamic interaction among flow rate, pressure drop and density (i.e. void fraction). When such parameters are included in a simplified (e.g. second order) ordinary differential equation based on an analogy with a mechanical vibration, the dynamics of flow is identified effectively by examining the damping term.[15] As is well-known, a self-sustained oscillation appears when the damping term is negative. So as to construct such simplified modeling, a boiling channel system, or related two-phase flow dynamics, must be formulated with only a few important parameters - the above-mentioned flow rate, pressure drop and void fraction. A typical example of such simplified modeling of the density wave oscillation is found in a paper by Hirayama et al.[16] They analyzed a natural-circulation parallel-channel system using a lumped-parameter approximation. They successfully included a void propagation delay in formulating the density wave oscillation, and derived a 3rd-order differential equation, which resulted in simplified stability criteria applicable to a wide range of parameters.

Among these parameters, the void fraction plays a key role in defining the flow transients. The pressure drop in two-phase flow is a function of mass flux and density. The mass flux is, in principle, uniform along the channel, because the compressibility of the fluid is rather small, even in the case of phase change. This is mainly because the propagation velocity of pressure wave is rather high compared with the flow velocity: the pressure wave travels at several tens meters per second in two-phase flow while two-phase mixture flows at only several meters per second. The flow instability is normally a slow transient, and thus the compressibility does not play a dominant role. So, the propagation delay of mass flux needs not be considered in formulating the slow transient. On the other hand, the density perturbation is transported at very low velocity - the same order of magnitude as the characteristic time or oscillation period of the density wave oscillation. Such density perturbation traveling along the channel is referred to as the "void wave" or "kinematic density wave"[17], which is different from the dynamic density wave propagated at the pressure wave velocity.

3. Problems in Continuous Flow Hypothesis

The void wave (disturbance in the void fraction) is a unique aspect of two-phase flow. It is well-known that a gas phase has, in general, higher velocity than a liquid phase, typically in a vertical upward flow. This is mainly because the buoyancy force is exerted on the gas phase relative to the surrounding liquid. When closely observed, in the flow, the gas bubbles move up and the liquid moves down alternatively - i.e. gas phases move up successively leaving the liquid phase behind. Such an instantaneous feature or topology of phases has been referred to as the "flow pattern" or "flow regime". In other words, the flow regime is like a still-image photograph: a frozen state of phase distribution. This is a very important feature in understanding the two-phase flow behavior. The phase distribution is not a fixed one but fluctuates in time, especially in the transient state. As discussed above, conventional modeling, including the void wave theory by Zuber et al.[17], is, however, based on continuous flow hypothesis by averaging in time and space. It should be further noted that constitutive relationships used in the simulation have been specified in the respective flow regime. Thus the flow regime must be identified in the course of simulating the transient state. These constitutive relationships are discontinuous over the flow regime boundary. Hence, in the two-fluid modeling, a certain buffer zone, similar to the transition zone, between flow regimes is set so as not to suffer from a discontinuity of the variables. When introducing an interfacial area concentration and interfacial area transport equation, the two-fluid modeling is pronounced to be free from such flow regime identification,[18] while additional parameters such as bubble size and bubble number density are differ drastically between the flow regimes. The bubble size in bubbly and slug flows are a typical example.

Suppose a large bubble expansion in the vertical liquid column, as in the work by Fukano et al.[19] The flow field initially consists of still water above a large compressed gas bubble. With longitudinal expansion of the gas bubble, the liquid above the bubble is pushed upward and is ejected from the tube end. Liquid phase surrounding the expanding bubble moves down toward the bubble tail. The bubble tail, then, moves upward by accumulating liquid shed from the bubble nose. In this bubble tail region, a significant wake flow appears, and the gas is entrained from the body of the bubble into the wake. The flow regime encountered in this case is single-phase liquid flow, a bubbly flow in the wake, the body of a large bubble, and so on. Such a flow field is considered as one of the features of the void wave. When analyzing such a flow field, it may not be suitable to use continuous flow modeling - e.g. two-fluid model, even with the interfacial transport equation - because such a model does not premise local, instantaneous features of the flow. In other words, in such a transient situation, the validity of the statistically-obtained mean values is not ensured. The same can be said for the interfacial area concentration. In analyzing such transient behavior, it may be better not to include too much structural detail, as is usual in two-fluid modeling. If we focus too much detail on the flow, we may miss the principal feature of transients. It is best to avoid such a situation: not seeing the forest for the trees. To avoid such problems, it may be necessary to develop a new alternative two-phase flow modeling of void wave, including only a few essential parameters or substantial mechanisms so as to be applicable universally to all two-phase flow phenomena.

4. Void Wave Related to Heat Transfer in Slug and Churn Flows

A channel flow is significantly constrained by the wall - the fluids flow essentially along the wall, and the heat may be added through the wall. The heat transfer problem is normally limited to the near-wall region, even in the case of single phase flow. The fluid friction, or frictional pressure drop, is due to a shear force found there. With the exception of an inverted annular flow or film boiling, the liquid is located adjacent to the wall. Thus two-phase friction and heat transfer are problems of the liquid phase and the wall.

Suppose a whole boiling channel system - from subcooled boiling to dryout. In the subcooled boiling, nucleate boiling dominates the heat transfer. The nucleate boiling is independent of the void fraction but is a weak function of the velocity. In low quality saturated boiling, the nucleate boiling dominates as well. Thus, from a macroscopic standpoint, the heat transfer is seen to be determined only by the heat flux. A typical example is the correlation of the Jens-Lottes[20] equation for nucleate boiling flow. With an increase in vapor quality, flow regime changes from a bubbly flow into a slug flow and then a churn flow. Finally an annular flow appears. When the liquid film adjacent to the wall is rather thin and the velocity of the gas core rather high in the annular flow, heat transfer is determined not only by the film thickness but also by the gas velocity. Thus, both gas phase velocity and void fraction may be of prime importance when discussing heat transfer.

In the bubbly and annular flows, the void fraction distribution along the channel is rather uniform and there exist no serious problems in estimating the heat transfer and pressure drop using conventional models. In the slug and churn flows, however, both the void fraction and the liquid phase distribution show a large fluctuation along the axis of the channel. In the liquid slug, sufficient liquid exists adjacent to the wall. On the contrary, in the preceding (or succeeding) large bubble section, only a thin liquid film exists around the bubble. When the heat flux is sufficiently high, the liquid film around the bubble may dry out. When applying modeling based on statistically-averaged parameters, two-fluid modeling or any other modeling based on averaging in time and space, such dryout phenomena of the liquid film around the bubble and the pressure (drop) fluctuation are rather hard to realize in the numerical simulation. If the whole boiling channel is focused, not only bubbly and annular flows, but also slug and churn flows, which are normally difficult to deal with, may be, even roughly, included in the numerical analysis or simulation, keeping their principal features. In other words, we must emphasize the importance of including somehow the local and temporal fluctuation of void fraction (i.e. the void wave) in analyzing thermal flow dynamics of two-phase flow.

5. Additional Problems to be Considered in Two-Phase Flow Dynamics Modeling

In addition to the flow problems, we should also consider the wall effect, especially in the context of heat transfer problems. In conventional boiling heat transfer research, wall heat capacity was normally ignored because of the steady state assumption, even in the case of the critical heat flux (CHF) in the slug flow region. Especially in the downward flow, bubbles generated suffer from the buoyancy effect, which impedes smooth removal of the bubble from the heated wall. In such a case, the flow is unstable and fluctuates. Thus the wall heat capacity may play a role, to a certain extent, in the CHF or film dryout around the stagnated bubbles. In the two-fluid modeling, such phenomena are hardly simulated because of the statistically-averaged model. If we want to analyze such phenomena, we are able to use so-called "regime-based" modeling in which there is a specified geometrical configuration of the interface based on the frozen image of the flow. Is the modeling really suitable and does it provide relevant information about the thermal flow field? Even if answer is "Yes", the next problem is how to include the wall heat capacity. In our experiment, the wall heat capacity or wall thickness had considerable influence on the CHF data. This infers the problem in the accuracy of the wall temperature measurement. Normally, wall temperature is measured at the outer wall, and the data suffers from phase delay and gain reduction. This is especially serious in the transient state.

During the oscillatory condition, the thermal penetration depth limits the wall effect. Beyond this thickness, being more pronounced in a conventional tube, measured data will not directly reflect the inner wall condition. In simulating such a case, it may be necessary to include thermal diffusion in a radial direction throughout the tube wall. This is not an easy task. As one of solution for avoiding such difficulty, a very thin-walled tube may be used.

The thermal flow field in the transients is rather complicated both in cross-section and along the tube. If the amplitude of oscillation is large enough, a fluid parcel is drawn back upstream. This is pushed forward during the next phase of oscillation. During the course of such process, the fluid parcel is, of course, heated, and becomes to have much higher enthalpy than expected from linear distribution along the channel. And it is not just the enthalpy: the same may be said for the void fraction. In other words, it is essential to trace the fluid parcel in the Lagrangian frame of reference, or somehow to include the history of fluid parcel movement. This is mainly because the local phenomena are not determined there but rather under influences of upstream or downstream, where the fluid parcel is located before the time of event. This is also not an easy task. In summarizing the above-mentioned discussion, the two-phase thermal flow dynamics should be analyzed in a framework based on void wave propagation, history, and wall-heat capacity surrounding the flow.

6. New Approach to Two-Phase Flow Dynamics

Based on the above discussion, we will now discuss a new approach to two-phase flow dynamics. The heat transfer problem will not be discussed here. In analyzing thermal flow problems in a boiling channel, conservation equations of mass, momentum and energy should be solved simultaneously. However, this is, in reality, rather hard because of the properties of momentum conservation equation. Provided a uniform pressure distribution and/or a predetermined flow velocity at the channel inlet, the momentum conservation equation is not necessarily solved. Instead, we see that the mass conservation is of prime importance. We assume here that the dominant flow field is determined by the mass balance. The void propagation equation is, in reality, based on the mass conservation. The energy equation in the boiling region is represented by the vapor generation or evaporation rate, when thermodynamic equilibrium is assumed. The generated vapor is considered as a source term in the mass conservation of gas phase, while in the liquid phase the generated vapor is taken into account as a sink term. In cases of isothermal flow field, the field equation is given only by the void propagation equation with several constitutive relationships.

The continuous flow modeling is not used here, because the continuous flow is based on statistically-averaged properties. The present aim is to provide locally-meaningful and, moreover, well-suited modeling to the analysis of the transient behavior of two-phase flow. Thereafter locally-detailed mechanisms - such as bubble size, number density, interfacial area concentration and interfacial waves, which are taken into account in two-fluid modeling - are ignored, but the relevant properties are included, as discussed below.

This modeling is based on a frame of reference using a length scale suitable for discussion of the principal features. For example, in order to discuss the void fraction fluctuation accompanying slug flow, a length scale comparable to the tube diameter would be the maximum size. In this case, the length scale is set at the tube diameter d - i.e. the frame of reference is the cylindrical volume of d in diameter and d in length. In order to realize the void fluctuation of such a volume, only one cylindrical bubble is provided with d_b in diameter and d_b in length. Then the void fraction of the volume is given by a simple function of d and d_b. The bubble provided here is a hypothetical bubble representing multiple bubbles, or a bubble swarm. Then the representative bubble size d_b, corresponding to bubbly flow, becomes much smaller than the length scale d, while the slug flow and annular flow are represented by a bubble train with rather large bubble diameters slightly less than the tube diameter. To calculate this bubble movement, it is essential to introduce the force balance or momentum balance exerted on the bubble, even under a quasi-steady assumption. Once this bubble movement has been traced, the void fluctuation may be given locally and instantaneously, but with a spatial resolution of tube diameter.

7. Principal Mechanisms to be Included

The present modeling aims at the reconstruction of realistic void fluctuation using the above-mentioned hypothetical bubble model. The physics to be considered begins with the mass conservation or void propagation equation. Thereafter, the momentum effect should be considered - both as bubble movement relative to liquid motion and as the compression or expansion of the gas phase due to static pressure change. The bubble velocity in a stagnant liquid is calculated from the balance between the drag force and the buoyancy force. Once determined, this speed is referred to as the terminal velocity. In the same way as the drift-flux model, the bubble velocity is then given by adding the velocity of center of volume, determined by the volume balance, to the terminal velocity. In addition, the velocity field is modified by the wake velocity induced by the succeeding bubble. This wake velocity brings about bubble acceleration and agglomeration of the bubbles as observed in the experiments. The second mechanism, compressibility, is introduced so as to realize a large-scale bubble expansion under transients. In reality, the compressibility has a clear effect on both large-scale and small-scale expansion. The third mechanism is directly related to this bubble expansion in the confined channel geometry: gas expansion is compensated within certain multiple frames of reference. When the expanded bubble volume exceeds the reference volume, this excess must be redistributed into the neighboring reference volumes located in the flow direction along the tube. This hypothetical, but realistic mechanism is referred to, in this paper, as the "phase redistribution due to geometrical constraint".

The numerical simulation for the void propagation is conducted using only a limited number of field equations, together with an additional small number of principal mechanisms mentioned above. Such an approach is, in general, referred to as "pattern dynamics". A typical example of pattern dynamics is the Lorenz model.[21] Another example is so-called 'cellular automata' (CA) simulation. In CA simulation, we normally use a metaphor model, which is different from the present modeling in the sense that the present model is more realistic and physically meaningful than the CA simulation.

The numerical simulation was conducted by applying a finite-difference scheme. The simulation results are not quoted here, but some typical application examples can be found elsewhere.[22,23] The void fluctuations obtained in the simulation demonstrated probability density functions (PDF) similar to those of bubbly, slug, churn and annular flows, respectively. Identified flow regimes based on this PDF coincide well with the well-known Mishima-Ishii diagram.[24] The time-averaged value of void fraction coincides equally well with the well-known drift-flux model.[17] In addition, the pressure drop agrees with the traditional Lockhart-Martinelli correlation.[25]

Besides the present modeling, regime-based modeling also gives void fluctuation similar to the present pattern dynamics. In regime-based modeling,[19, 26, 27] the flow regime is specified, so that the time-evolution of geometrical configuration of interface is limited within a definite range - i.e. the topology of interface is in principle consistent throughout the transients. Thus, the transition of flow regime is hardly realized in the simulation. This is the main difference from the newly proposed modeling, where the flow regime is not specified at all. In this sense, the present pattern dynamics modeling is much more substantial than regime-based modeling.

8. Future Perspective

The present model has a high potential for simulating both local and axial void fraction fluctuation in a two-phase flow system, even in the case of flow reversal. So, present pattern dynamics modeling is extended in application so as to meet the problems in boiling two-phase flow and related CHF, especially in slug and churn flow regimes; in flow instabilities such as geysering, including rapidly-expanding large bubbles due to self-evaporation or rapid decompression; in vertical downward flow; and in horizontal flow. In order to simulate such flow fields, it is, of course, essential to obtain the correlation for wake velocity and geometrical constraints or phase redistribution mechanisms, especially in the case of horizontal flow.

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