536.423.1:532.55

Paper No. 167-15

# Pressure Drops in $CO_2$ Supercritical Boilers*

## 2nd Report Dynamic Characteristics of Frictional Pressure Drop

By Tadashi SAKAGUCHI** Koji AKAGAWA**

Mamoru OZAWA*** Takafumi ORIGANE****

Experimental and analytical investigations were conducted on the dynamic behaviors of the frictional pressure drop in a $CO_2$ forced flow heated loop at supercritical and subcritical pressures near the critical pressure. The step responses of the pressure drop to the step changes of inlet flow rate, heat flux and exit pressure were measured. The analytical results obtained by using a smsll perturbation method and Laplace transformation agreed approximately with the experimental results.

### 1. Introduction

Characteristics of transient behavior of the pressure drop in a once-through steam generator is one of the important factor to be considered in the design. And sufficient information on the transient behavior is required for determining the stability margins of a steam generator.

In the preceding papers[1,2] the experimental results on the transient behaviors of the pressure drop in a low pressure water loop and comparisons with a theoretical analysis were reported. And also the static characteristics of the frictional pressure drop in supercritical pressure region which was investigated in a $CO_2$ supercritical boiler were reported in the 1st report[3].

The present paper presents the experimental and analytical results on the transient behavior of the frictional pressure drop with the changes of the inlet flow rate, the heat flux and the exit pressure in supercritical and subcritical steam generators.

Carbon dioxide is used as a working fluid in this experiments, and thus this paper may contribute to a steam power plant design as well as to a $CO_2$ power plant design[4].

### 2. Analysis

There are two methods available for analysing the transient behavior of the frictional pressure drop in a once-through steam generator; a nonlinear numerical analysis such as a finite difference method and a linear analysis with a small perturbation method and Laplace transformation. The former method has a wide validity, but it is difficult to get the physical meaning of the phenomena for the calculated results. therefore, in this paper the transient behavior is analysed with the linear analysis by using a distributed parameter model with linear approximations of the conservation equations of mass, energy and momentum and the thermophysical properties which are similar to those in the papers of Sekoguchi et al.[5] and Shima et al.[6]

The experiments are conducted at 80 ata as a supercritical pressure. At this pressure the thermo-physical properties near the pseudo-critical point change sharply with the temperature as shown in Fig.1, where $H$ is the enthalpy and $T$ is the temperature.

The $H$-$T$ relation is approximated by three straight lines. The straight lines AB and CD mean a constant specific heat and the vertical BC means that the enthalpy increases at constant temperature as in the saturated condition. The region AB where the enthalpy is below the value $H'$ is referred to as the preheated region, the region CD where the

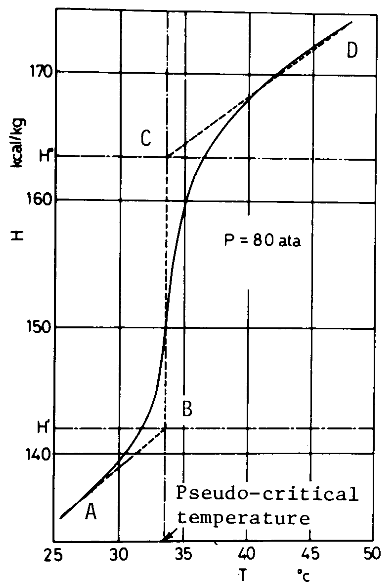


Fig.1 Thermo-physical properties

* Received 15th June, 1977.
** Professor, Faculty of Engineering, Kobe University, Kobe.
*** Research Assistant, Faculty of Engineering, Osaka University.
**** Engineer, Kawasaki Heavy Industry Co., Ltd.

enthalpy is above the value $H''$ is referred to as the superheated region and the intermediate region at the pseudo-critical temperature is referred to as the transition region. The enthalpies $H'$ and $H''$ correspond to the saturated liquid and vapor enthalpies at subcritical pressure respectively. The specific volumes $v'$ and $v''$ or the specific heats $c_p'$ and $c_p''$ at the states corresponding to $H'$ and $H''$ respectively are determined from the table of the thermo-physical properties of $CO_2$.

In order to formulate the transient behavior of the frictional pressure drop, the following assumptions are used.

(1) The thermo-physical properties don't change with the pressure change along the tube.

(2) The specific volumes in the preheated and the superheated regions are constant and are equal to $v'$ and $v''$ respectively. In the transition region the specific volume is a linear function of the enthalpy.

(3) The flow is one dimensional.

(4) The heat capacity of the tube wall in the transition region (boiling region) can be neglected.

(5) The friction factor is constant along the heated tube.

(6) The flow in the transition region is homogeneous.

(7) The kinetic energy is negligible in comparison with the thermal energy.

Under these assumptions the equations derived for the transient behavior of the frictional pressure drop in the previous paper[1] can be applied to those at supercritical pressure.

The frictional pressure drop in the whole heated tube can be written as follows;

$$\Delta P_f=\int_0^{Z_B}\frac{\lambda}{2g_r D}\left(\frac{G}{F}\right)^2 v' dZ+\int_{Z_B}^{Z_s}\frac{\lambda}{2g_r D}\left(\frac{G}{F}\right)^2 v dZ+\int_{Z_s}^{L_0}\frac{\lambda}{2g_r D}\left(\frac{G}{F}\right)^2 v'' dZ \qquad \cdots\cdots\cdots\cdots(1)$$

where

$G$ : flow rate,
$g_r$ : gravitational acceleration,
$D$ : inner diameter of the heated tube,
$F$ : cross sectional area of the heated tube,
$Z_B$, $Z_s$ : coordinate of the inlet and the exit of the transition region respectively,
$\lambda$ : friction factor and
$L_o$ : heated length.

Each term in the right hand side of Eq.(1) corresponds to the frictional pressure drop in the preheated, the transition and the superheated regions respectively. The first two terms were already formulated in the preceding paper[1], and the third term is formulated in this paper.

The third term in the right hand side of Eq.(1) is linearised and Laplace transformed, then the frictional pressure drop perturbation is expressed by

$$\overline{\delta P}_{f3}=-\frac{\lambda}{2g_r D}\left(\frac{G_0}{F}\right)^2 v''\overline{\delta Z}_s+\frac{\lambda}{g_r D}\left(\frac{G_0}{F}\right)^2 v''\bar{g}_3 L_{30} \qquad \cdots\cdots\cdots\cdots(2)$$

where $\delta P_{f3}$, $\delta G_3(=G_o g_3)$ and $\delta Z_s$ are the perturbations in the frictional pressure drop, the flow rate and the location of the boundary between the transition region and the superheated region respectively, and subscripts 3 and $\circ$ and superscript $-$ represent the superheated region, the steady state values and the Laplace transformed values respectively.

The first term in Eq.(2) represents the perturbation in the frictional pressure drop due to a shift of the boundary between the transition region and the superheated region. The second term represents the perturbation in the frictional pressure drop due to a perturbation in the flow rate at the inlet of the superheated region. In determining the pressure drop response in the whole heated tube, the first term in Eq.(2) is cancelled by the corresponding term in the transition (boiling) region. Therefore, in the following section the first term is omitted from the equation of the perturbation in the frictional pressure drop.

Here, the perturbation in the flow rate at the inlet of the superheated region $\bar{g}_3$ which is equal to that at the exit of the transition (boiling) region is expressed by the following equation which was obtained by Terano[7],

$$\bar{g}_3=\bar{q}+\left\{\frac{(1+\alpha)^{-T_e s}}{T_e s-1}-\frac{T_e s}{T_e s-1}\frac{1}{1+\alpha}\right\}(\bar{q}-\bar{g}_i) \qquad \cdots\cdots\cdots\cdots(3)$$

where

$q$ : dimensionless perturbation in the heat flux $=\delta Q/Q_o$ ,
$s$ : parameter of Laplace transformation,
$\alpha = v''/v'-1$ ,
$T_e = F(H''-H')/\{Q_o \pi D(v''-v')\}$,

and subscript $i$ represents the value at the inlet of the heated tube.

Then the frictional pressure drop response to the input $q$ or $g_i$ of the first order lag with the time constant $T_R$ is expressed by

$0 \leqq t \leqq \tau_2$

$$\frac{\delta P_{f3}}{\Delta P_0}=\frac{L_{30}}{L_0}\left[2(1+\alpha)(1-e^{-t/T_R})q-\frac{2T_e}{T_e+T_R}(e^{t/T_e}-e^{-t/T_R})(q-g_i)\right] \qquad \cdots\cdots(4)$$

$\tau_2 \leqq t$

$$\frac{\delta P_{f3}}{\Delta P_0}=\frac{L_{30}}{L_0}\left[2(1+\alpha)(g_i-qe^{-t/T_R})+\frac{2e^{-t/T_R}}{T_e+T_R}\{T_e+T_R(1+\alpha)^{T_e/T_R+1}\}(q-g_i)\right] \qquad \cdots\cdots\cdots\cdots(5)$$

The responses during the period $0 \leqq t \leqq \tau_2$, $0 \leqq t \leqq \tau_c$ in the preheated and the transition (boiling) regions are expressed by Eqs.(6) and (7) respectively. Those during the other periods are presented in the preceding paper[1], hence omitted in this paper.

$$\frac{\delta P_{f1}}{\Delta P_0}=\frac{L_{10}}{L_0}\left\{2(1-e^{-t/T_R})g_i-\frac{1}{\tau_c}\left[t-\frac{1}{T_{kw1}-T_R}\{T_{kw1}^2(1-e^{-t/T_{kw1}})-T_R^2(1-e^{-t/T_R})\}\right](q-g_i)\right\} \qquad \cdots\cdots\cdots\cdots(6)$$

$$\frac{\delta P_{f2}}{\Delta P_0} = \frac{L_{20}}{L_0}\left\{\frac{1+\alpha}{\tau_e}\frac{L_{10}}{L_{20}}\left[t - \frac{1}{T_{kw1}-T_R}\right.\right.$$
$$\left.\times \{T_{kw1}^2(1-e^{-t/T_{kw1}}) - T_R^2(1-e^{-t/T_R})\}\right]$$
$$\times (q-g_i) - \left[\frac{T_e}{T_e+T_R}e^{t/T_e} - \frac{2T_e+T_R}{T_e+T_R}e^{-t/T_R}\right.$$
$$+1-\frac{1}{\alpha}\left\{\frac{1}{2} - \frac{T_R^2 e^{-t/T_R}}{(T_e+T_R)(T_e+2T_R)}\right.$$
$$\left.\left. - \frac{T_e}{T_e+T_R}e^{t/T_e} + \frac{T_e}{2(T_e+2T_R)}e^{2t/T_e}\right\}\right]$$
$$\left.\times (q-g_i) + (2+\alpha)(1-e^{-t/T_R})q\right] \cdots\cdots\cdots (7)$$

where

$\Delta P_o = \lambda(G_o/F)^2 v' L_o/(2g_r D)$,
$\tau_c = \tau_1(1+T_{k1}/T_{w1})$,
$t$ : time,
$\tau_1$ : residence time in the preheated region $= L_{1o}F/(G_o v')$,
$\tau_2$ : residence time in the transition region $= T_e \ln(1+\alpha)$,
$T_{k1} = \gamma_B c_B (D_o^2 - D^2)/(4h_1 D)$,
$T_{w1} = D c_p'/(4h_1 v')$,
$T_{kw1} = (1/T_{k1} + 1/T_{w1})^{-1}$,
$c_B$ : specific heat of the tube material,
$D_o$ : outer diameter of the heated tube,
$h$ : heat transfer coefficient and

subscripts 1 and 2 represent the preheated and the transition regions respectively. In this analysis, the same assumptions are used in the preheated and the superheated regions, then the characteristic time constants in the superheated region can be defined in the same manner as in the preheated region.

$$\tau_s = \tau_3(1+T_{k3}/T_{w3})$$
$$\tau_3 = (L_0 - Z_{s0})F/(G_0 v'')$$
$$T_{k3} = \gamma_B c_B(D_0^2 - D^2)/(4h_3 D)$$
$$T_{w3} = Dc_p''/(4h_3 v'')$$

## 3. Experimental apparatus and experimental procedures

The experimental apparatus used in this study is the same as the one in the previous paper[3]. It consists of a plunger pump, a preheater, a test section and a condensor. The test section is an SUS304 spiral tube with 3.95 mm I.D., 7.0 mm O.D., 28.3 m length, 246 mm diameter of the spiral and 1.35 m height, and is heated directly by AC power. The heat flux is uniform because of the constant wall thickness of the test tube.

The pressure drop in the test section is measured by two pressure transducers, a bridge circuit and an amplifier, and is recorded by a pen-oscillograph; and also the flow rate at the test section inlet, the exit pressure and the fluid temperature at the test section exit are recorded simultaneously.

The transient response of the pressure drop to a step change of the inlet flow rate is measured keeping the other parameters such as the inlet temperature, the heat flux and the exit pressure constant. The same procedure is taken about the step changes of the heat flux or the exit pressure.

## 4. Experimental results

The gravitational and the accelerational pressure drops are very small compared with the frictional pressure drop and they are less than 3 % of the frictional pressure drop in the steady or the transient conditions in this experiment. Therefore, the measured transient response of the pressure drop to the flow rate or the heat flux changes can be considered with sufficient accuracy to be that of the frictional pressure drop. But the accelerational component is dominant in the transient response of the pressure drop to the exit pressure change.

In the following section, the perturbation of the pressure drop $\delta P_f$ or $\delta P$ is normarized by dividing $\Delta P_o$ and also by the dimensionless input cahnge $g_i=(G_o'-G_o)/G_o$, $q=(Q_o'-Q_o)/Q_o$ and $p=(P_o'-P_o)/P_o$ , and it is shown against the dimensionless time $t/T_e$, where $G_o'$, $Q_o'$ and $P_o'$ are the value at the new steady state after the transient conditions.

The steady state value of the frictional pressure drop $\Delta P_{fo}$, the characteristic time constants $\tau_c$, $\tau_2$, $\tau_s$ and $T_e$ are shown in Table 1, where the notation $T_i$ represents the fluid temperature at the test section inlet.

### 4.1 Frictional pressure drop response to the step change of the inlet flow rate

The experimental results at supercritical pressure 80 ata are shown in Fig.2 for each value of $g_i$ as a parameter. For any value of $g_i$ , the frictional pressure drops increase rapidly at first and take a maximum value. Then the pressure drops decrease

Table 1 Initial steady state and characteristic time constant

| $P_o$ ata | $G_o$ kg/h | $Q_o$ kcal/m$^2$h | $T_i$ °c | $\Delta P_{fo}$ kg/cm$^2$ | $\tau_c$ s | $\tau_2$ s | $\tau_s$ s | $T_e$ s | $\Delta P_o$ kg/cm$^2$ |
|---|---|---|---|---|---|---|---|---|---|
| 80 | 70 | 1.14×10$^4$ | 22 | 6.96 | 5.21 | 2.94 | 10.5 | 3.97 | 3.98 |
| | | 0.58×10$^4$ | 22 | 4.48 | 10.4 | 5.21 | — | 7.94 | 3.98 |
| 65 | 70 | 1.14×10$^4$ | 22 | 8.25 | 2.18 | 3.62 | 21.5 | 3.26 | 3.53 |
| | | 0.58×10$^4$ | 22 | 4.97 | 4.35 | 6.67 | — | 6.53 | 3.53 |
| 60 | 70 | 1.14×10$^4$ | 22 | 8.65 | 1.05 | 3.79 | 28.4 | 3.15 | 3.36 |
| | | 0.91×10$^4$ | 22 | 6.98 | 1.31 | 4.54 | 17.9 | 3.77 | 3.36 |
| | | 0.58×10$^4$ | 22 | 5.27 | 2.09 | 6.94 | — | 6.30 | 3.36 |

gradually and finally settle to a new steady state. The characteristics of the response curves are similar to the experimental results with low pressure water(2).

The experimental results at subcritical pressure 65 ata are shown in Fig.3. The tendency of the response curves is similar to that at supercritical pressure, but the curves at subcritical pressure have smaller overshoots and longer settling times than those at supercritical pressure.

The analytical results calculated with various values of $T_R$ are also shown in Figs.2 and 3 comparing with the experimental results. In the case of $T_R$=0 , the frictional pressure drop increases instantaneously, increases exponentially and then settles to a new steady state. In the case of $T_R$=0.5, the analytical result agrees with the experimental results in the early stage of transition.

The characteristic time constant $\tau_s$ is larger than the values $\tau_c$ or $\tau_2$ in this experiment as is shown in Table 1. Therefore, the perturbation in the frictional pressure drop due to the variation in the specific volume in the superheated region changes more slowly than the frictional pressure drop response in the preheated and the transition region. This simple analysis does not take into account the variation of the specific volume in the superheated region, so the analytical results have less overshoots than the experimental results.

The settling time $T_s$ and the peak time $T_p$ which are normalized by $T_e$ are plotted against the reduced pressure $P_o/P_{cr}$ in Fig. 4, where $T_s$ is defined as the time when the frictional pressure drop takes the value of $\pm$ 5% of the new steady state value on the response curve as shown in Fig.2 and $T_p$ is defined as the time when the frictional pressure drop takes the maximum value.

The absolute value of $T_s$ or $T_p$ depends on the operating conditions. But in the case of constant values of $G_o$ , $Q_o$ and $T_i$ and in the range $0.046 \leq g_i \leq 0.248$ , , the dimensionless characteristic time $T_s/T_e$ or $T_p/T_e$ decreases with an increase in the reduced pressure. This shows that the flow in the heated tube approaches the non-compressible fluid flow with an increase in the system pressure and that the time delay of the response decreases with an increase in the system pressure.

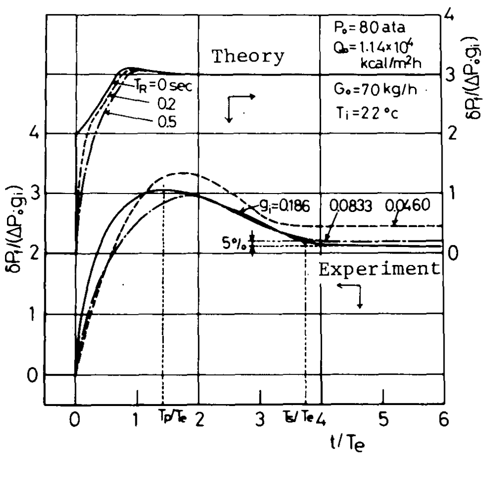


Fig.2 Frictional pressure drop response

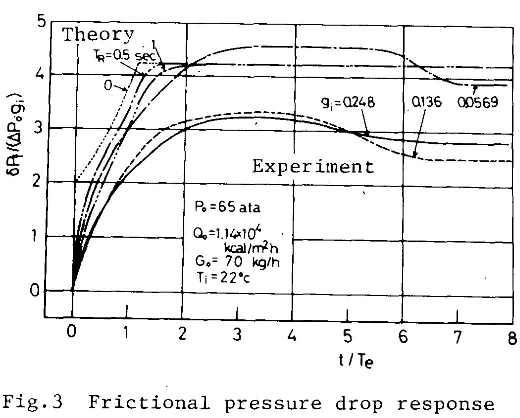


Fig.3 Frictional pressure drop response

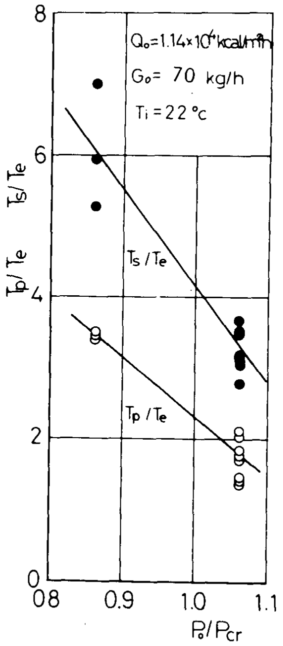


Fig.4 Settling time and peak time

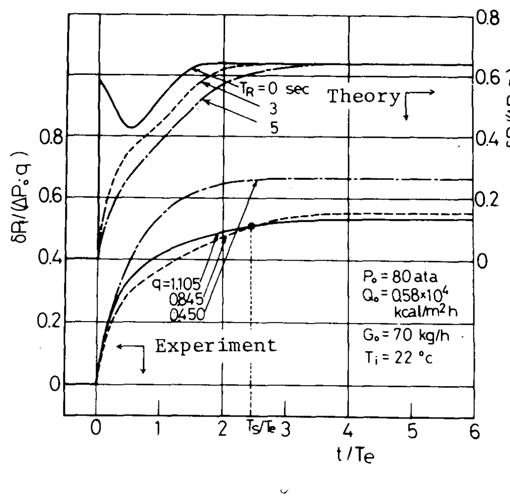


Fig.5 Frictional pressure drop response

4.2 Frictional pressure drop response to the step change of the heat flux

The experimental results on the frictional pressure drop response at $P_o$=80 ata to the step increase and to the step decrease in the heat flux are shown against $t/T_e$ in Figs.5 and 6 respectively. It can be seen from these figures that the tendencies of the response curves for $q$>0 and for $q$<0 are different from each other and that the settling time in the former case is shorter than in the latter case.

The response curves in Fig.5 are close to a first order lag. On the other hand, the frictional pressure drops decrease with higher rate at first and then with low rate and finally settle to the new steady state in Fig.6, where the perturbation terms $\delta P_f$ and the input changes $q$ are negative and thus the values $\delta P_f/(\Delta P_o \cdot q)$ are positive.

The experimental results of the frictional pressure drop responses at subcritical pressure $P_o$=65 ata are shown in Figs.7 and 8. In the case of $q$>0 (Fig.7) the response curves are close to a first order lag. On the other hand, in the case of $q$ < 0 (Fig.8) each response curve is composed of three parts: rapid decreaseing part, constant part and gradual decreasing part.

The experimental results at $P_o$=60 ata are shown in Figs. 9 and 10. In the case of $q$>0 (Fig.9) the response curves are close to a first order lag as in the case of $P_o$=80 or 65 ata. In the case of $q$ <0 (Fig.10) the response curves are close to

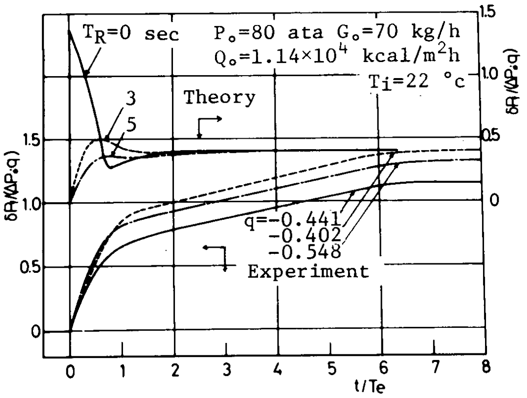


Fig.6 Frictional pressure drop response

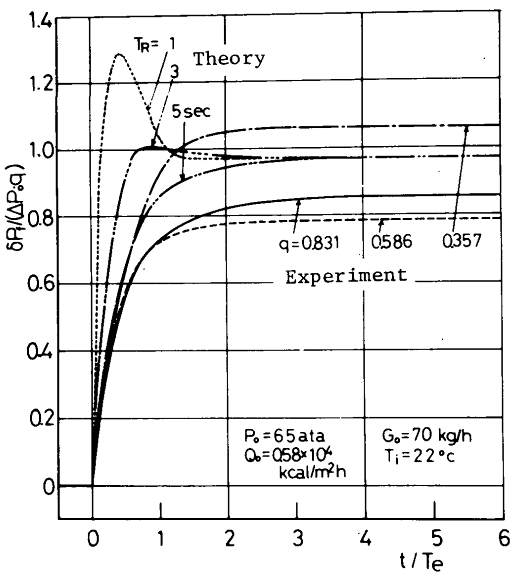


Fig.7 Frictional pressure drop response

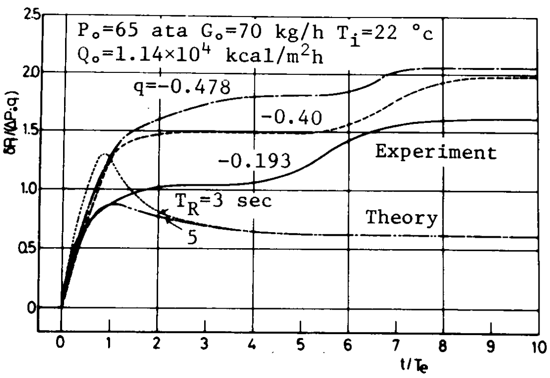


Fig.8 Frictional pressure drop response

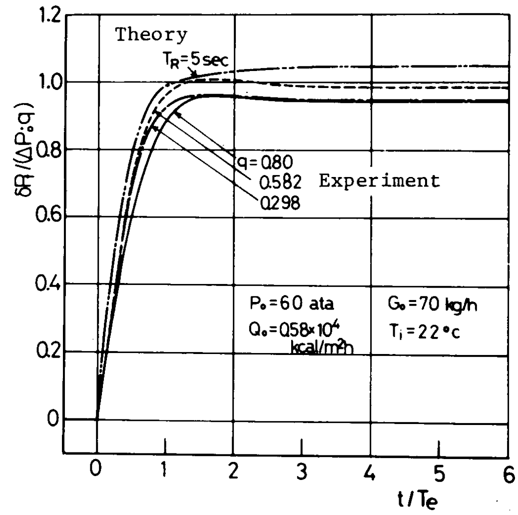


Fig.9 Frictional pressure drop response

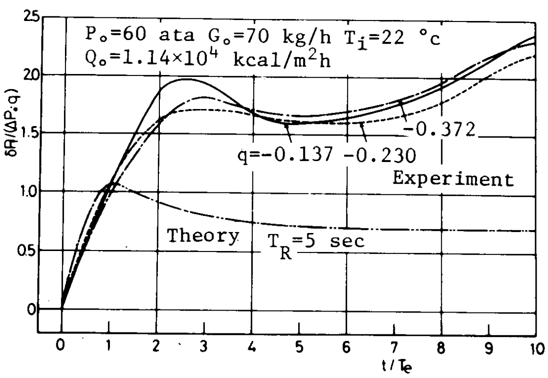


Fig.10 Frictional pressure drop response

those in Fig.8. The characteristics of the response curves for $q > 0$ and for $q < 0$ at subcritical pressure are also different from each other as in the supercritical pressure region.

The difference of the characteristics of the response curves may be caused by the difference of the initial stady state condition; in the case of $q > 0$, the system is composed of tow regions, i.e., the preheated and the transition (boiling) regions, and in the case of $q < 0$, the system is composed of three regions, i.e., the preheated, the transition (boiling) and the superheated regions.

The analytical results are also shown in Figs.5-10 comparing with the experimental results. In the case of $T_R=0$(Figs.5 and 6), the response curves have peaks at the initial stage of the transition, and the agreement with the experimental results is not good. The analytical results with a first order lag, $T_R$=5 sec, agree well with the experimental ones for $q > 0$. This time constant $T_R$=5 sec is larger than that of the heat capacity of the tube wall $T_k$=1 - 1.5sec. So this large time delay may be caused by some mechanism which is not taken into account in this simple analysis.

On the other hand, in the case of $q < 0$ the analytical results with $T_R$=5 sec do not agree with the experimental results except in the early stage of the transition. This may be due to the linear approximations of the conservation equations and the assumption that the specific volume in the superheated region is constant.

The settling time $T_s$ is normalized by $T_e$ and is plotted against the reduced pressure in Fig.11. The value $T_s/T_e$ increases with an increase in $Po/P_{cr}$ in the case of $q > 0$, on the contrary the value $T_s/T_e$ decreases with an increase in $Po/P_{cr}$ in the case of $q < 0$ . In order to explain this phenomenon more detailed investigation on the flow in the heated tube is required.

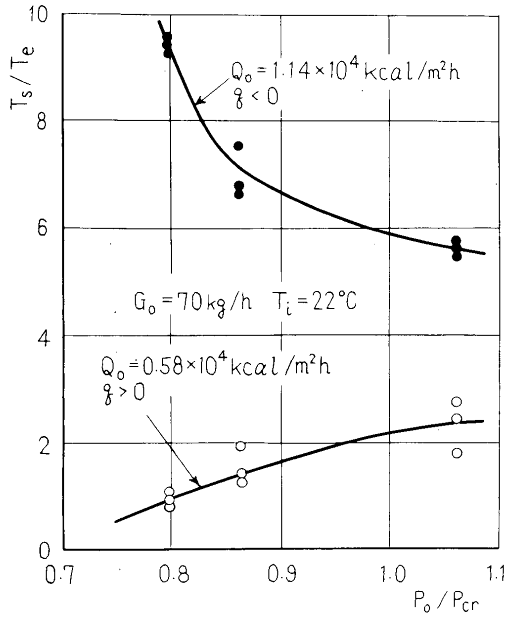


Fig.11 Settling time

4.3 Pressure drop response to the step change of the exit pressure

The pressure drop response to the step change of the exit pressure corresponds to the transient behavior of the inlet pressure of the test section in this experiment. This transient behavior is concerned with the blowdown problem in a nuclear power plant.

The experimental results at $P_o$=80 ata are shown in Fig.12. The pressure drop increases instantly and then decreases like a first order lag. The tendencies of the response curves at $P_o$=65 and 60 ata or those with low pressure water[8] are similar to that shown in Fig.12.

The initial large peak on the response curve is due to a temporary increase in the flow rate caused by the expansion of the fluid. Suppose an ideal step decrease in the exit pressure, the point A in Fig.12 is determined as the inital point and the response curve is approximated by a first order lag. The time constant $T_d$ which represents the speed of the response is defined as the time when the pressure drop decreases by the amount of 63.5 % of the

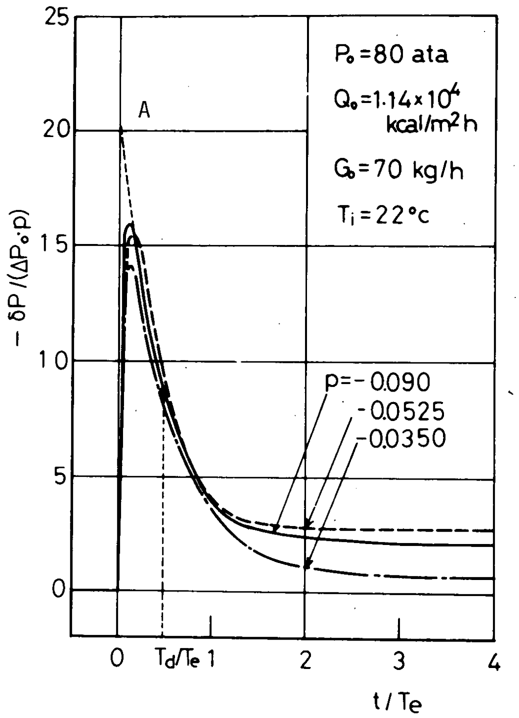


Fig.12 Pressure drop response

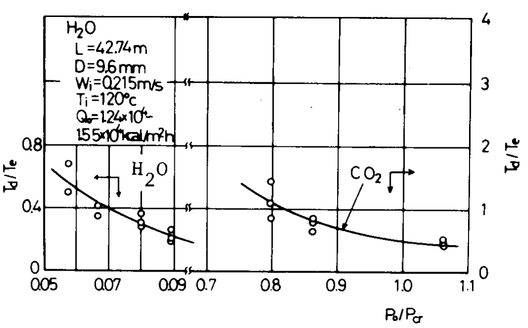


Fig.13 Time constant of first order lag

difference between the state A and the new steady state.

The time constant $T_d$ obtained in this $CO_2$ experiment and the experiment with low pressure water[8] are normalized by $T_e$ and plotted against the reduced pressure $P_o/P_{cr}$ in Fig.13. The value $T_d/T_e$ decreases with an increase in $P_o/P_{cr}$. This tendency may be caused by the fact that the variation of the specific volume along the tube decreases with an increase in the system pressure.

## 5. Conclusions

An experimental study on the transient behavior of the frictional pressure drop is conducted by using a $CO_2$ forced flow heated loop at supercritical and subcritical pressures, and the transient behavior of the pressure drop is analysed by a small perturbation method and Laplace transformation.

The conclusions of this study are summarized as follows;

(1) The response curve of the frictional pressure drop to the step change of the inlet flow rate has an overshoot. The settling time and the peak time decrease with an increase in the reduced pressure.

(2) The response curve of the frictional pressure drop to the step increase in the heat flux is close to a first order lag at any system pressure, but the characteristics of the response curve to the step decrease in the heat flux are different from those to the step increase in the heat flux.

(3) The response curve of the pressure drop to the step decrease in the exit pressure has a peak. The time constant of the response curve decreases with an increase in the reduced pressure.

(4) The characteristics of these transient behaviors (1), (2) and (3) are similar to those in the low pressure water experiment[2,8].

(5) The analytical results to the input with a first order lag agree with the experimental results for the step increase in the flow rate and the heat flux.

Authors wish to express thanks to Messrs. M. Imot, K. Ogura and S. Nakai for their assistance in conducting the experiment. A part of this experiment was supported by the Fund of Scientific Research granted by the Ministry of Education.

## References

(1) Akagawa, K., Sakaguchi, T., Bull. JSME, 14-77(1971), 1199.
(2) Akagawa, K., Sakaguchi, T., Bull. JSME, 15-80(1972), 264.
(3) Sakaguchi, T., et al., Bull. JSME, 20-143(1977), 644.
(4) Fujii, T., et al., Preprint of Japan Soc. Mech. Engrs. (in Japanese), 760-19 (1976), 171.
(5) Sekoguchi, K., Sakai, M., Trans. Japan Soc. Mech. Engrs., 32-240(1966), 1249.
(6) Shima, M., et al., Trans. Japan Soc. Mech. Engrs., 34-262(1968), 1126.
(7) Terano, T., Monthly Reports of Transportation Technical Research Institute, 7-10(1957), 1.
(8) Akagawa, K., Sakaguchi, T., Preprint of Japan Soc. Mech. Engrs. (in Japanese), 795th (1969), 31.