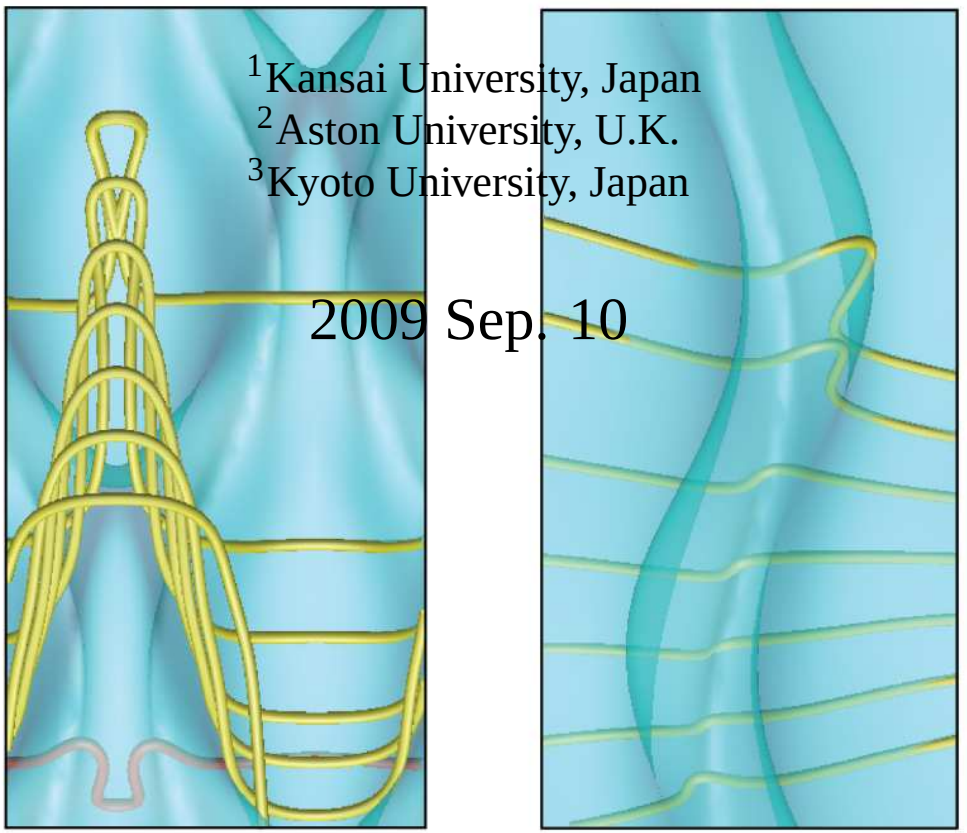
Symmetry of Coherent Vortices in Plane Couette Flow

T. Itano[1], S. C. Generalis[2], S. Toh[3], J. P. Fletcher[2]

[1]Kansai University, Japan
[2]Aston University, U.K.
[3]Kyoto University, Japan

2009 Sep. 10



Hairpin-shaped vortex

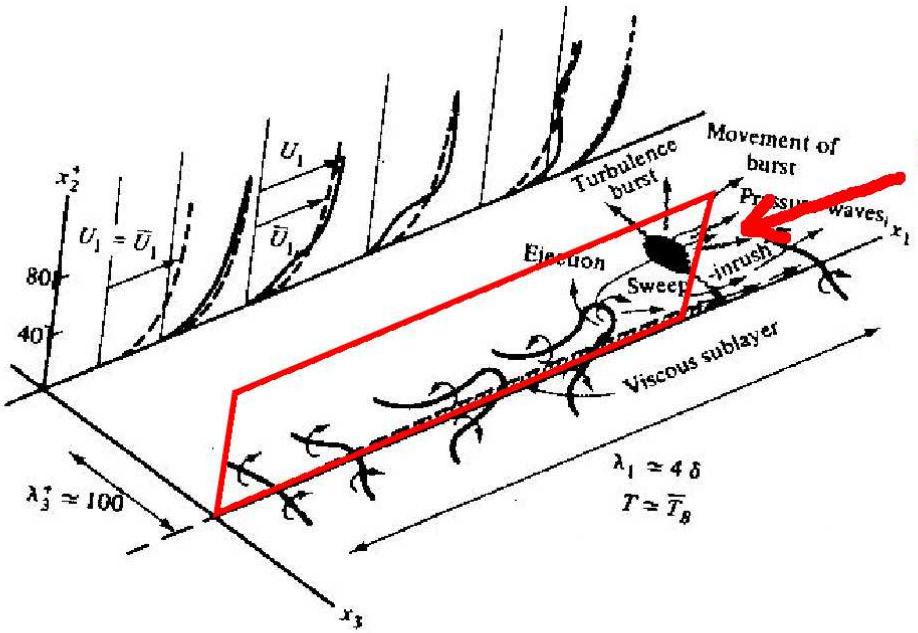
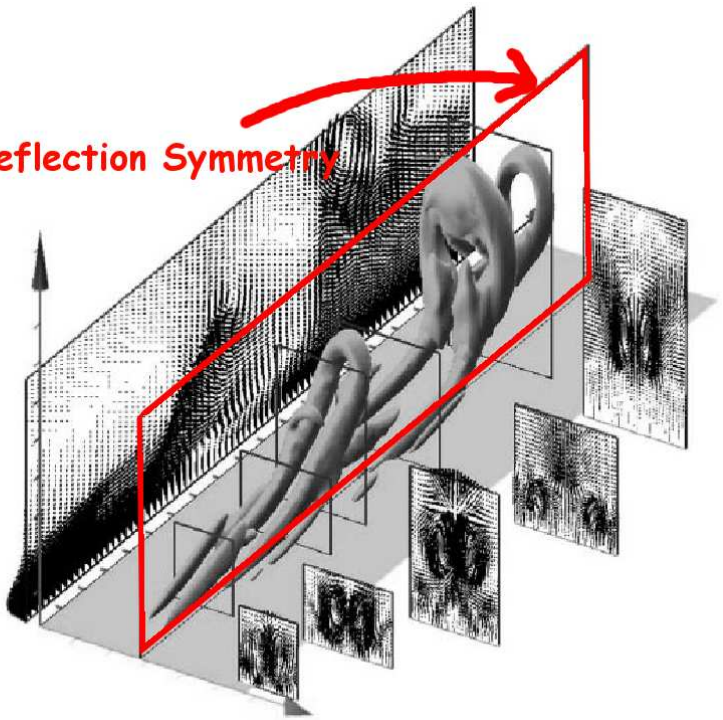


Figure 9 Conceptual model of the turbulence near the wall during a cyclic process, with average spacings λ_1 and λ_2 (from Hinze 1975).



Hinze(1975)

Adrian(2000)

1. one of predominant structures in wall-bounded flows
2. modification of shape / reflection symmetry in common

Motivation

1. Exact solution of plane Couette flow (PCF)

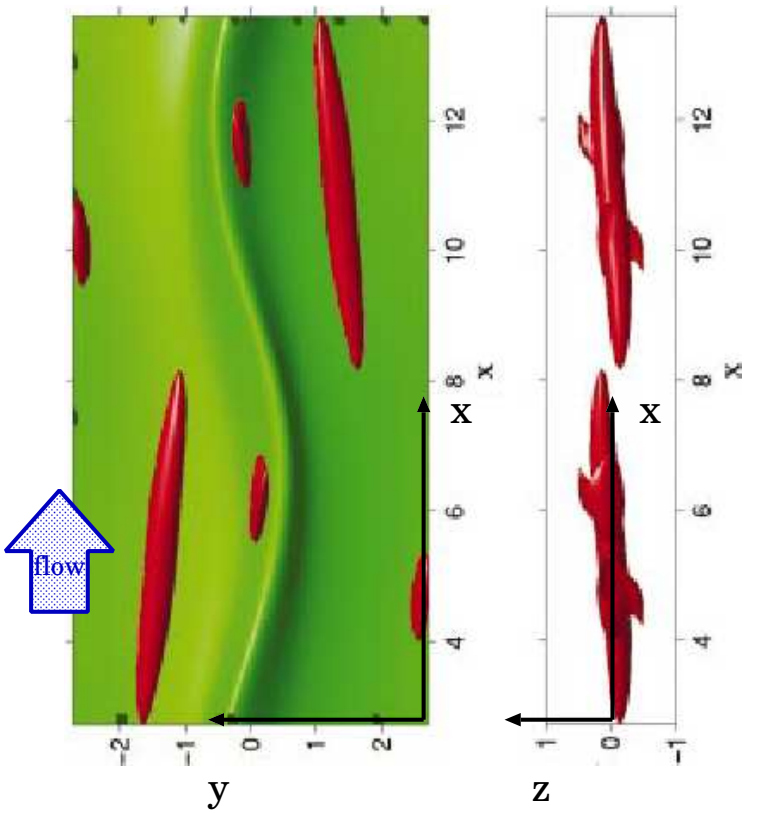
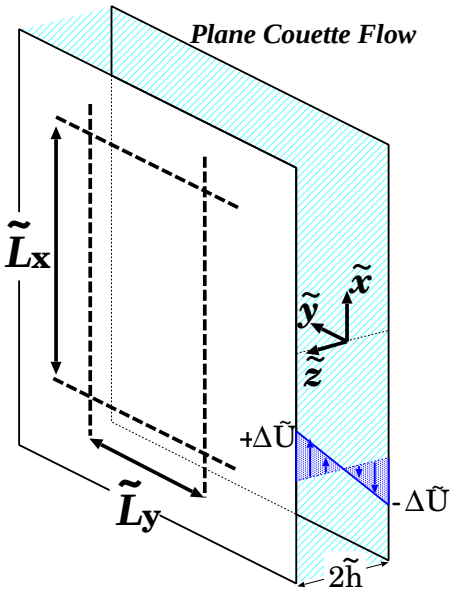


Figure: Waleffe (2003), meandering streak, $Re_{\min} = 127$

hairpin-vortex type solution exists in PCF ?

Formulation

1. configuration



2. steady / nondimensionalised equation

$$\boldsymbol{\nabla}\cdot\boldsymbol{u}=0\,,\quad(\boldsymbol{u}\cdot\boldsymbol{\nabla})\boldsymbol{u}=-\boldsymbol{\nabla}p+\frac{1}{Re}\boldsymbol{\nabla}^2\boldsymbol{u}\,,$$

$$\boldsymbol{u}(x,y,z=\pm1)=\pm\boldsymbol{e}_x,$$

periodic b.c. in x(streamwise), y(spanwise) directions.

3. Galerkin-method with Fourier and modified Chebyshev expansion

historically : The well-known solution of PCF

- Nagata(1990), Clever & Busse(1997), Waleffe(1998) → "NBW"
- Kawahara & Kida(2001) : periodic sol. embedded in turbulence
- Gibson et al.(2008), Halcrow et al. (2009)

for comparison re-calculation, one vortex line = a hairpin ?, explot !

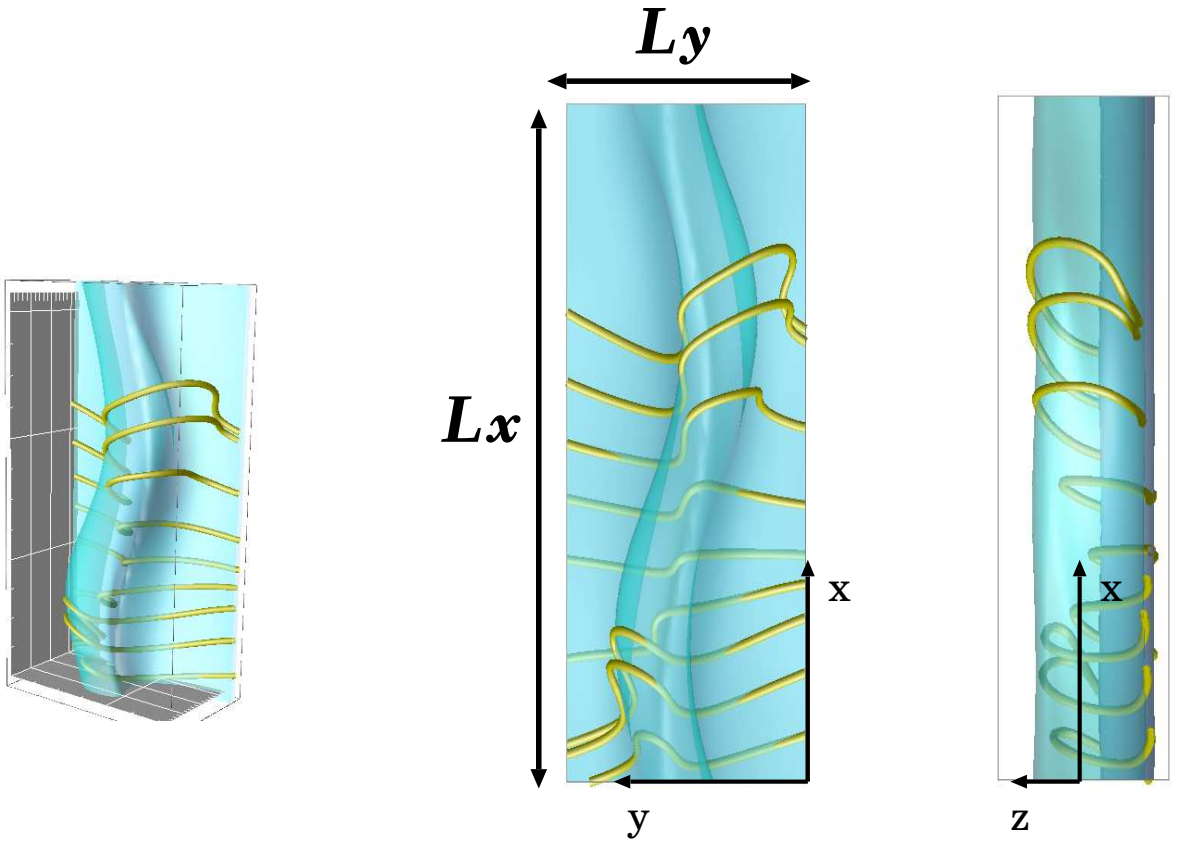


Figure: NBW at $Re = 200$ (u.b.)

new result : Hairpin vortex solution (HVS)

- Schmiegel(1999)
- I. and Generalis(2009)
- Gibson et al.(2009)

reflection symmetry → hairpin (bundle of vortex lines)

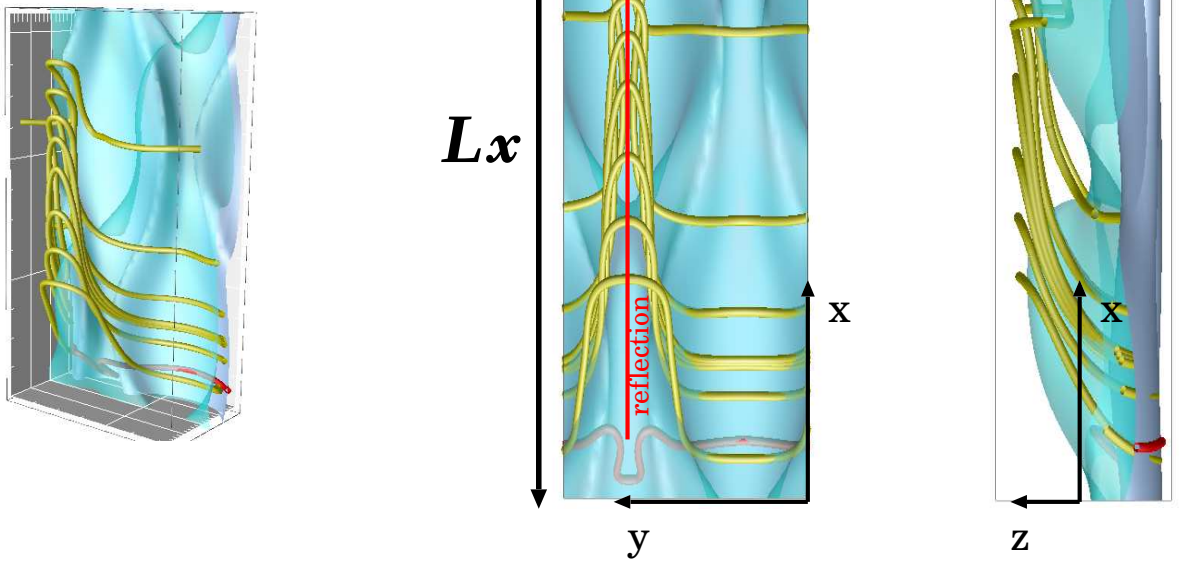
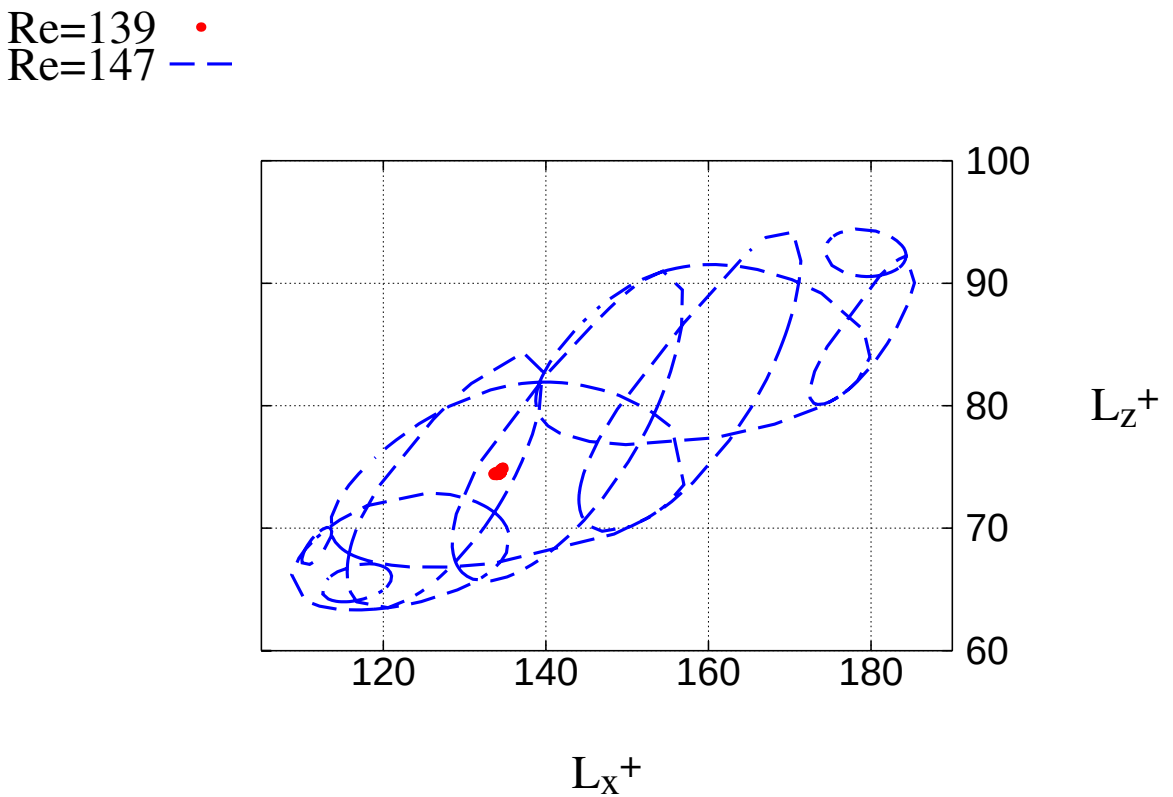


Figure: HVS at $Re = 200$ (u.b.)

#1. Streamwise and spanwise extents in wall units

streak characteristic spanwise extent $L_z^+ = 100$?



1. $L_z^+ \approx 75$ at the minimum Reynolds number $Re_{\min} = 139$
2. Island HVS exists abruptly spreads out with Re increasing

#2. Multiple scales in HVS

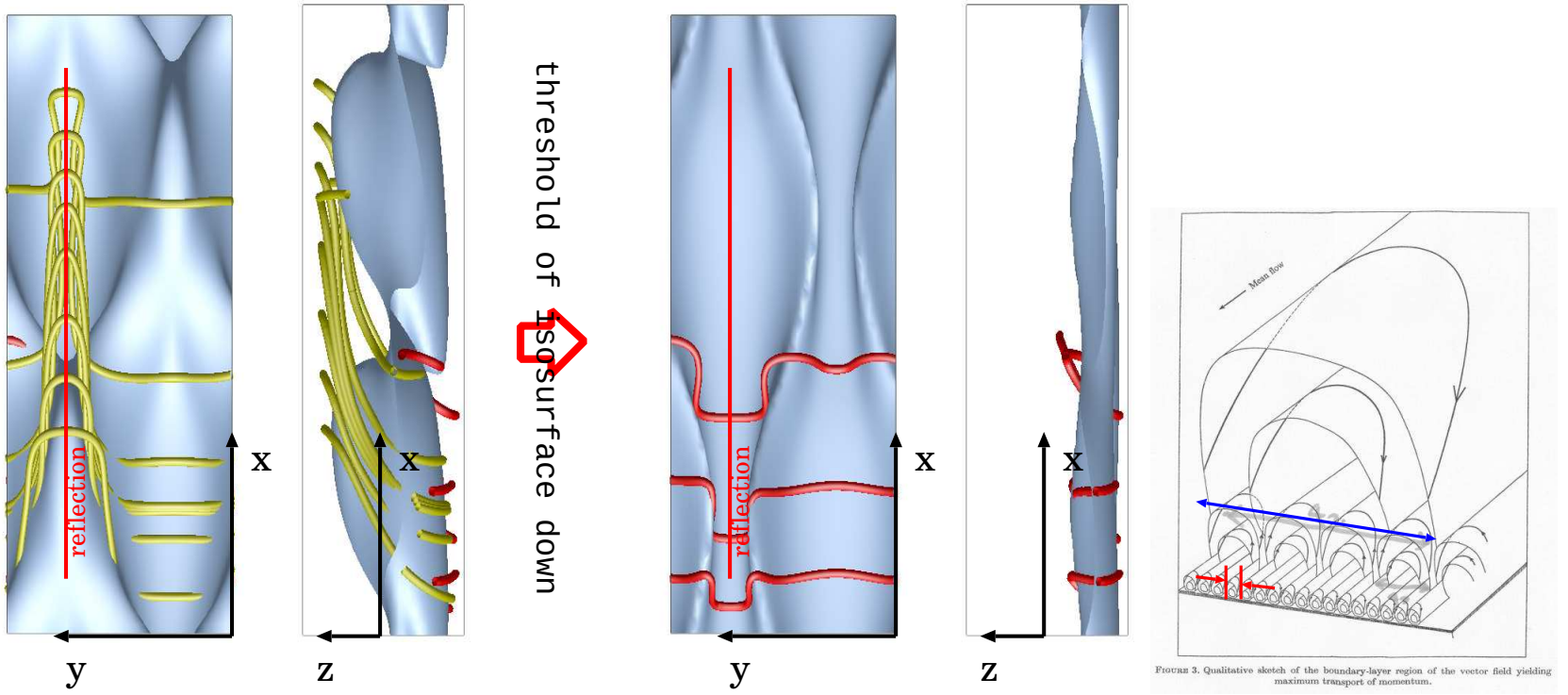


Figure: (left) $u = -0.1$ a single kink $\rightarrow$ $u = -0.4$ two kinks ,
(right) Busse (1970)

related to hierarchy of structures in turbulence ?

#3. Bifurcation of NBW from HVS

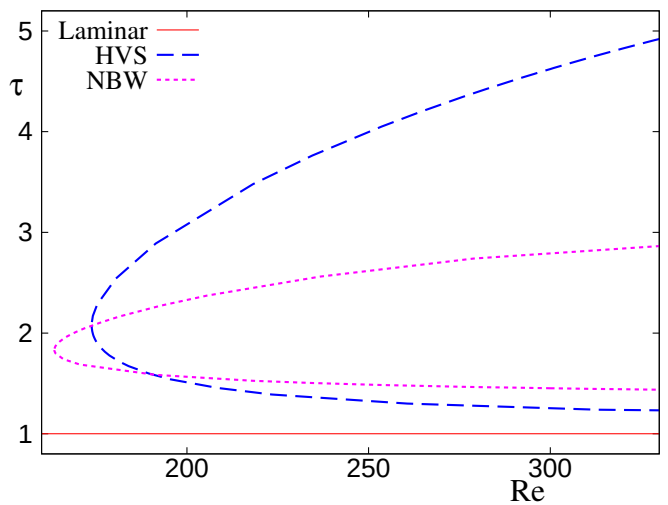
Extending the system with a parameter ϵ $(0 \leq \epsilon \leq 1)$

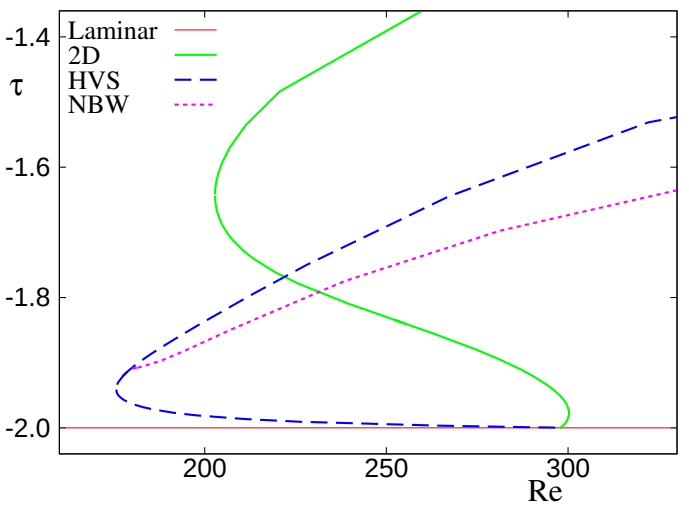
$$\nabla \cdot \boldsymbol{u} = 0 ,$$

$$(\boldsymbol{u} \cdot \nabla)\boldsymbol{u} = -\nabla p + \frac{6\epsilon}{Re} z \boldsymbol{e}_x + \frac{1}{Re}\nabla^2 \boldsymbol{u} .$$

$$\boldsymbol{u}(x, y, z = \pm 1) = \pm(1 - \epsilon)\boldsymbol{e}_x .$$

$$\text{PCF} : \epsilon = 0 \quad \rightarrow \quad \epsilon = 1$$





NBW (quaternary branch,pink) bifurcates from HVS (tertiary branch,blue) at $\epsilon = 1$ via the breaking of a reflection symmetry.

Summary

1. hairpin vortex solution of PCF
2. down to $Re_{\min} = 139$ in high resolution, $L_z^+ = 75$
3. Scale hierarchy of structures in HVS
4. NBW (derivative) bifurcates from HVS (primitive)

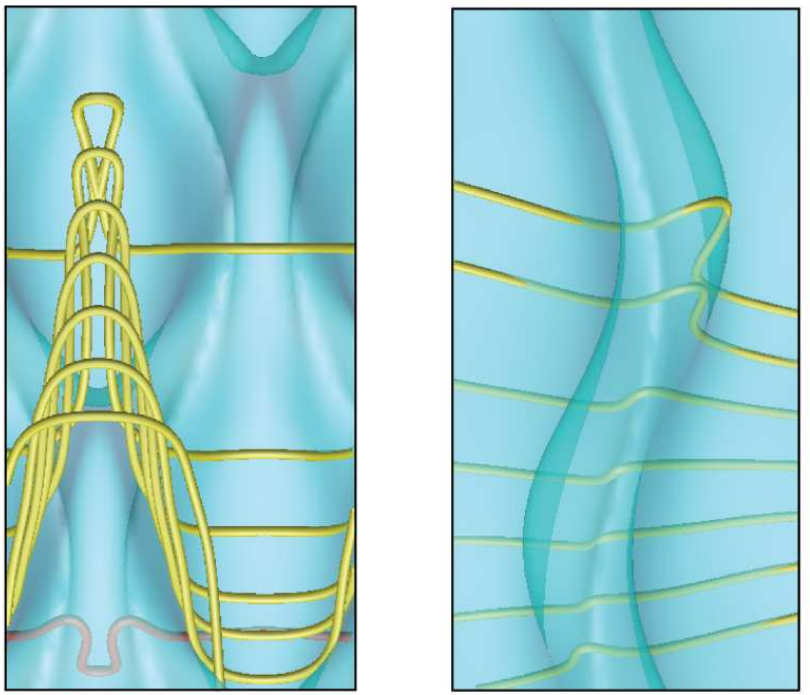


Figure: left: HVS , right: NBW

T. I. and S.C. Generalis, Phys. Rev. Lett., **102**, 114501 (2009)