Emission Regulation and Long-Run Optimality under Imperfect Monitoring

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The long-run optimality conditions are examined. It will be shown that input taxes will not achieve them. For the application of input taxes, a transfer payment is necessary. The optimal tax schemes are derived under imperfect monitoring. A pure emission tax, a pigouvian tax, fails to achieve the long-run optimality conditions. Introductions of an output tax and a transfer payment are necessary for the long-run optimality.

Keywords: Emission Regulation, Imperfect Monitoring, Long Run Optimality, Pigouvian Tax

1. Introduction

Impositions of pigouvian taxes on emissions have been widely accepted as suitable remedies for externalities. In a word of certainty and perfect monitoring, pigouvian taxes actually achieve social optimality conditions by themselves. Neither other type taxes nor transfer payments are necessary. These properties are showed by Spulber (1985). On the other hand, in a world of uncertainty and imperfect monitoring, these properties may not be hold. For instance, Schmutzler and Goulder (1997) show that the combinations of an output tax and an emission tax would derive higher social welfare in some circumstances.

The purpose of this paper is to extend the analyses of emission regulation in a world of uncertainty and imperfect monitoring. We will discuss the validity of various tax schemes in this framework. More specifically, we will examine the validity of tax schemes by comparing their consequences with the long-run optimality conditions. The long-run optimality conditions are important issues. It is because appropriate tax schemes in the short-run do not necessarily achieve the long run optimality conditions. It is possible appropriate tax schemes in the short run fail to provide correct entry incen-
tives to the firms. In such circumstances, transfer payments are necessary to induce socially optimal entry (see Carlson and Loury 1980).

The paper is organized as follows. In Section 2, we define a regulator's problem under perfect monitoring. A regulator's object is to choose industry output, inputs, and the number of firms so as to maximize social welfare. For this purpose, the regulator has to achieve two necessary conditions. They are the input combination condition and the entry condition. The input combination condition tells whether the firms are using inputs appropriately. The entry condition, alternatively, states a necessary adjustment in a long-run profit condition. To maximize social welfare, both conditions are necessary. In Section 3, we show emission taxes will achieve both conditions but input taxes will achieve neither of them. In Section 4, we will extend the above analysis under imperfect monitoring. We will derive the input combination condition and the entry condition under imperfect monitoring. In Section 5, the validity of tax schemes will be examined with these conditions. Then, we will show necessary adjustments for maximizing social welfare. Section 6 presents conclusions and policy implications.

2. The Social Optimum under Perfect Monitoring

Although the ultimate object of this paper is to analyze the social optimality conditions under imperfect monitoring, we first introduce the basic model under perfect monitoring. The model introduced here is the similar model as Spulber (1985).

There are $n$ identical firms producing a homogeneous output $q$. These firms use inputs $x_j$ given input prices $r_j$, $j=1,\cdots,m$. The firm's production technology is characterized by the production function, $q=f(x_1,\cdots,x_m)\equiv f(X)$. $f$ is twice differentiable, increasing and concave. In the course of production, firms generate emissions, $e$, as a by-product. The emission level is determined by the pollution function, $e=h(x_1,\cdots,x_m)\equiv h(X)$. $h$ is also twice differentiable. The sign of its first derivative is positive but the sign of its second derivative is unknown. Emissions are pollutants and cause social welfare loss. The reduction of social welfare is captured by the social damage function $D(E)$ where $E=ne$. The social damage function is assumed to be differentiable $dD(E)/dE\equiv D'(E)\equiv D'(ne)>0$ and $D(0)=0$.

The regulator's problem is to choose the industry output, emission, inputs, and the number of firms so as to maximize social welfare subject to
the technological constraints. Let \( P(*) \) be the inverse demand function and \( F \) be a fixed cost. Then, the Lagrangian for the regulator is

\[
L = \int_0^{nq} P(s) ds - n \sum_{j=1}^m r_j x_j - nF - D(ne) + n\xi(f(X) - q) + n\delta(e - h(X)) \quad (1)
\]

where \( \xi \) and \( \delta \) are the shadow prices for the production and emission constraints, respectively. The first-order necessary conditions include

\[
\frac{\partial L}{\partial q} : \quad P(nq) - \xi = 0 \quad (2)
\]

\[
\frac{\partial L}{\partial e} : \quad -D'(ne) + \delta = 0 \quad (3)
\]

\[
\frac{\partial L}{\partial x_j} : \quad -r_j + \xi \frac{\partial f(X)}{\partial x_j} - \delta \frac{\partial h(X)}{\partial x_j} = 0 \quad j = 1, \ldots, m \quad (4)
\]

\[
\frac{\partial L}{\partial n} : \quad P(nq)q - \sum_{j=1}^m r_j x_j - F - D'(ne)e = 0. \quad (5)
\]

Denote the optimal allocation and shadow prices \((q^*, X^*, e^*, \xi^*, \delta^*)\). Then, combining the first-order conditions, \( q = f(X) \) and \( e = h(X) \), we will have the optimality condition of the input combinations.

\[
P(n^*q^*) \frac{\partial f(X^*)}{\partial x_j} = r_j + D'(n^*e^*) \frac{\partial h(X^*)}{\partial x_j} \quad j = 1, \ldots, m \quad (6)
\]

The marginal revenue product of each input should equal the marginal factor cost to society, the sum of the input price and the social damage in production activity. As Spulber pointed out, the principal optimality conditions are (5) and (6). The first-order condition (6) is the input combination condition. Once the appropriate input combination is chosen, both the output and emission levels are automatically optimized. Therefore, the first-order condition (6) will suffice for (3) and (4). The first-order condition (5) is the long-run entry condition. The number of firms in the industry must be determined after subtracting the social damage of emissions from the zero profit condition.
3. Regulation Under Perfect Monitoring

**Emission Tax**

If the firm behaves in a competitive manner, the firm takes output price, $P$, as given. Then, the Lagrangian for the firm's problem becomes

$$\ell = Pq - \sum_{j=1}^{m} r_j x_j - F - t_e e + \lambda(f(X) - q) + \sigma(e - h(X))$$

(7)

where $t_e$ is an emission tax. $\lambda$ and $\sigma$ are the shadow prices for the production and effluent constraints, respectively. The first-order necessary conditions include

$$\frac{\partial L}{\partial q} : P - \lambda = 0$$

(8)

$$\frac{\partial L}{\partial e} : - t_e + \sigma = 0$$

(9)

$$\frac{\partial L}{\partial x_j} : - r_j + \lambda \frac{\partial f(X)}{\partial x_j} - \sigma \frac{\partial h(X)}{\partial x_j} = 0 \quad j = 1, \ldots, m.$$  

(10)

Combining the first-order necessary conditions, we will have the firm's input combination conditions.

$$P \frac{\partial f(X)}{\partial x_j} = r_j + t_e \frac{\partial h(X)}{\partial x_j} \quad j = 1, \ldots, m.$$  

(11)

Setting the emission tax $t_e^* = D'(n^*e^*)$, the input combination conditions of equation (6) will be achieved. The optimal emission tax induces the firm to choose the appropriate input combination. With the emission tax $t_e^* = D'(n^*e^*)$, the total tax payment will be $D'(n^*e^*)e^*$. It will achieve the correct entry condition of equation (5).

**Input Taxes**

As we have seen, emission taxes will achieve both the input combination condition and the entry condition. Can we derive the same optimality conditions with other tax schemes? Spulber showed that output taxes would not achieve the optimality conditions. In this section, we will show that input taxes will not achieve the optimality conditions, either.
Denote the input taxes \( t_j, \ j = 1, \cdots, m \). Then, the firm’s short-run profit maximization problem is

\[
\begin{align*}
\text{Max} = Pf(X) - \sum_{j=1}^{m}(1 + t_j)r_jx_j.
\end{align*}
\]  

(12)

The first-order conditions for the firm is then

\[
\begin{align*}
Pr\frac{df(X)}{dx_j} = r_j + t_jr_j & \quad j = 1, \cdots, m.
\end{align*}
\]  

(13)

Thus, the firm chooses the input combination so as to satisfy

\[
\begin{align*}
\frac{\partial f(X)}{\partial x_i} &= r_i + t_i, \\
\frac{\partial f(X)}{\partial x_j} &= r_j + t_jr_j.
\end{align*}
\]  

(14)

From equation (6), the appropriate input combination conditions need to satisfy

\[
\begin{align*}
\frac{\partial f(X^*)}{\partial x_j} = r_j + D'(n^e)(\partial h(X^*)/\partial x_j) \\
\frac{\partial f(X^*)}{\partial x_i} &= r_i + D'(n^e)(\partial h(X^*)/\partial x_i) \quad i \neq j; \ i, j = 1, \cdots, m.
\end{align*}
\]  

(15)

Therefore, in order to induce the firm to adopt the appropriate input combinations, the regulator needs to set the input taxes to its social damage,

\[
t_j = \frac{1}{r_j}D'(n^e)(\partial h(X^*)/\partial x_j) \quad j = 1, \cdots, m.
\]  

(16)

For this purpose, the regulator requires the knowledge about pollution productions of all firms. However, such knowledge is unlikely obtained.

**Proposition 1.** In order to derive the optimal input combination condition, the regulator has to know a pollution generation mechanism of each firm. Unless the regulator has the knowledge, the optimal input combination conditions can not be obtained.

When the regulator imposes input taxes, he or she rarely imposes input taxes on all inputs. In general, the regulator imposes inputs taxes only on several inputs. Here, we will see whether the regulator can achieve the social optimality conditions with such a partial input tax scheme. For the simplicity, suppose the firm uses only two inputs in the production and the regulator imposes an input tax on one of the two inputs. In order to induce
the firm to choose the appropriate input combination, the regulator has to adjust the input tax of \( t_1 \) to the level

\[
\frac{\partial f(X^*)/\partial x_2}{\partial f(X^*)/\partial x_1} = \frac{r_2 + D'(n^*e^*)}{r_1 + D'(n^*e^*)} = \frac{r_2}{r_1 + t_1 r_1}.
\]

To satisfy the above condition, \( t_1 < \frac{1}{r_1} D'(n^*e^*) \frac{\partial h(X^*)}{\partial x_1} \). However, in order to reduce the amount of emissions to the socially optimal level, the regulator has to raise the rate of \( t_1 \). It requires \( t_1 > \frac{1}{r_1} D'(n^*e^*) \frac{\partial h(X^*)}{\partial x_1} \). Clearly, these two conditions are not satisfied simultaneously. Hence, the social optimality conditions will not be achieved with a partial input tax scheme.

**PROPOSITION 2.** The social optimality conditions will not be achieved with a partial input tax scheme.

With the optimal input tax scheme, \( t_j = \frac{1}{r_j} D'(n^*e^*) \frac{\partial h(X^*)}{\partial x_j} \), \( j = 1, \ldots, m \). Then, the entry condition becomes

\[
Pf(X^*) = F - \sum_{j=1}^{m} r_j x_j^* - D'(n^*e^*) \sum_{j=1}^{m} \left[ \frac{\partial h(X^*)}{\partial x_j} \right] x_j^* = 0.
\]

(17)

If the pollution function, \( h(X) \) is homogeneous of degree one, \( \sum_{j=1}^{m} \frac{\partial h(X)}{\partial x_j} x_j = h(X) = e \). Then, the entry condition will be identical with equation (5),

\[
Pf(X^*) = F - \sum_{j=1}^{m} r_j x_j^* - D'(n^*e^*)e^* = 0.
\]

(18)

Otherwise, the optimal input tax scheme alone cannot induce the optimal entry condition. Thus, a transfer payment is necessary for the long-run optimality.

In the case of a single input with the optimal input tax, the entry condition is

\[
P(n^g)f(X) - r_1 x_1 - F - D'(ne) \frac{\partial h(x_1)}{\partial x_1} x_1 = 0.
\]

To provide correct entry incentives to the firms, the entry equation has to be
\[ P(n^* q^*) f(x_1^*) - r_1 x_1^* - F - D'(n^* e^*) e^* = 0. \]

To satisfy this requirement, a lump-sum tax (or subsidy) per firm is required

\[ T^* = \left[ D'(n^* e^*) \frac{h(x_1^*)}{x_1^*} - D'(n^* e^*) \frac{\partial h(x_1^*)}{\partial x_1} \right] x_1^*. \] (19)

If the pollution function of \( h(x_1) \) is globally convex, the sign of the square bracket is negative. Hence a lump-sum subsidy is required. On the contrary, if the pollution function of \( h(x_1) \) is globally concave, the sign of the square bracket is positive, a lump-sum tax is required.

Figure 1 shows the above argument. If the pollution function is globally convex \( (h_{x_1^*} > 0) \), the slope of the pollution function (marginal product \( \partial h_{x_1}/\partial x_1 \)) is always larger than the slope of a ray from the origin (average product \( h(x_1)/x_1 \)). In this case, the social damage of input will be over-estimated and the total tax bill paid by individual firms may be too large. Hence, a lump-sum subsidy is required. On the other hand, if the pollution function is globally concave \( (h_{x_1^*} < 0) \), the slope of the pollution function, \( \partial h_{x_1}/\partial x_1 \), is always smaller than the slope of a ray from the origin, \( h(x_1)/x_1 \). In this case, the social damage of input will be under-estimated and the total tax bill paid by individual firms may be too small. However, in general, we do not know the shape of the pollution functions and cannot examine the necessary transfer mechanism.

**Figure 1 Input Tax and Required Transfer**

\[
\begin{align*}
h_{x_1^*} < 0 & \quad \forall x_1 \Rightarrow \partial h(x_1)/\partial x_1 < h(x_1)/x_1 \Rightarrow Tax \\
h_{x_1^*} > 0 & \quad \forall x_1 \Rightarrow \partial h(x_1)/\partial x_1 > h(x_1)/x_1 \Rightarrow Subsidy
\end{align*}
\]
PROPOSITION 3. For a single input with free entry firms, the entry condition will not be achieved with an input tax. A transfer payment is required to adjust the difference between the marginal private cost and the average private cost of pollution.

In this section, we evaluate an emission tax and input taxes based on the social optimality conditions: the input combination condition and the entry condition. We showed that the emission tax would satisfy both the input combination condition and the entry condition. Therefore, the social optimality can be achieved with the emission tax. On the other hand, input taxes satisfy neither the input combination condition nor the entry condition. Therefore, the social optimality cannot be achieved with input taxes. In the next section, we will extend our analysis to the case under imperfect monitoring.

4. The Social Optimum Under Imperfect Monitoring

The regulator often does some monitoring activities for examining the emission level. These activities include establishments of monitoring stations, occasional inspections to the firms, and a system of fines. In the rest of this paper, we assume these types of activities are mandatory for the regulator to induce the firms to change their behavior and the regulator has to spend some monitoring cost for these activities.

The regulator’s problem is to choose industry output, emission, inputs, and the number of firms to maximize social welfare subject to technological constraints and monitoring methods available. The Lagrangian for the regulator is

$$ L = \int_0^n P(s)ds - n \sum_{j=1}^m r_j x_j - nF - D(ne) - M(n,q,e) + n \xi (f(X) - q) + n \delta (e - h(X)) $$

where $M$ is the minimum monitoring costs needed to induce the emission level $e$ and the output level $q$ with the number of firms $n$. It is assumed that $\partial M(n,q,e)/\partial n \geq 0$, $\partial M(n,q,e)/\partial q \geq 0$, and $\partial M(n,q,e)/\partial e \leq 0$. The first inequality means that a necessary monitoring cost becomes larger as the number

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1. See Hardford (1978), Malik (1990), and Swierzbinski (1994). For the summary of discussions, see Cropper and Oates (1992) and Batthold (1994).
of firms increases. For instance, a monitoring cost for spot-check to the factories becomes larger as the number of firms increases. The second inequality means that a monitoring cost becomes larger as the output level increases. For instance, the regulator needs to examine outputs produced if toxic materials are contained in outputs. In such a circumstance, a necessary monitoring cost will increase as the output level increases. The third inequality means a necessary monitoring cost becomes smaller as the emission level increases. For instance, the regulator may need not to use sophisticated devices to detect a large amount of pollution. In such a circumstance, a necessary monitoring cost becomes smaller as the emission level increases.

Figure 2 shows the relationships among a number of firms $n$, an output level $q$, an emission level $e$, and a necessary monitoring cost $M$. Given the number of firms $n^0$ and the output level $q^0$, the necessary monitoring cost to achieve the emission level, $e^0$, is $0M^0$. (See point A.) The necessary monitoring cost to achieve the same emission level would increase as the output level increases. For example, if the output level becomes $q^1$, the necessary monitoring cost will be $0M^1$ with the same number of firms $n = n^0$. (It corresponds to the movement from point A to point B.) Under a certain level of output, the smaller the emission level, the larger the necessary monitoring cost would be. For example, the necessary monitoring cost to achieve the
emission level \( e^1 \) where \( e^0 < e^1 \) will be \( 0M^2 \) when the output level is \( q^0 \) and the number of firms is \( n^0 \). (Compare point C with point A). Changing the number of firms also has an impact on the necessary monitoring cost. As the number of firms increases, monitoring cost functions will shift outward. For example, if the number of firms becomes \( n^1 \), the necessary monitoring cost to achieve the emission level \( e^0 \) would be \( 0M^3 \) given the output level \( q^0 \). It implies the increase of the necessary monitoring cost. (See point D.)

From the first-order necessary conditions of (20),

\[
\frac{\partial L}{\partial q} : \quad P(nq) - \frac{1}{n} \frac{\partial M(n,q,e)}{\partial q} - \xi = 0
\]

(21)

\[
\frac{\partial L}{\partial e} : \quad -D'(ne) - \frac{1}{n} \frac{\partial M(n,q,e)}{\partial e} + \delta = 0
\]

(22)

\[
\frac{\partial L}{\partial x_j} : \quad -r_j + \xi \frac{\partial f(X)}{\partial x_j} - \delta \frac{\partial h(X)}{\partial x_j} = 0 \quad j = 1, \ldots, m
\]

(23)

\[
\frac{\partial L}{\partial n} : \quad P(nq)q - \sum_{j=1}^{m} r_j x_j - F - D'(ne)e - \frac{\partial M(n,q,e)}{\partial n} = 0.
\]

(24)

Denote the optimum allocation and shadow prices \( (q^*, X^*, e^*, \xi^*, \delta^*) \). Then, combining the first-order necessary conditions, \( q = f(X) \), and \( e = h(X) \), we will have the optimal input combination conditions under imperfect monitoring.

\[
\left[ P(n^*q^*) - \frac{1}{n^*} \frac{\partial M(n^*,q^*,e^*)}{\partial q} \right] \frac{\partial f(X^*)}{\partial x_j} = r_j + \left[ D'(n^*e^*) + \frac{1}{n^*} \frac{\partial M(n^*,q^*,e^*)}{\partial e} \right] \frac{\partial h(X^*)}{\partial x_j} \quad j = 1, \ldots, m
\]

(25)

Equation (25) contains two aspects. First, the use of additional input raises the output level. Then, the larger the output level, the larger the monitoring cost will be. The first square bracket in (25) reflects the increase of the monitoring cost, \((1/n^*) \partial M(n^*,q^*,e^*)/\partial q\). The marginal factor benefit to society has to be evaluated after subtracting the increase of the monitoring cost from the output price.

Second, the marginal factor cost to society has to include the reduction of the monitoring cost through the emission expansion. The increase of emissions raises the social damage, \( D'(n^*e^*) \). However, it reduces the neces-
sary monitoring cost, \( \partial M(n^*, q^*, e^*)/\partial e \). The second square bracket in (25) contains these two counter effects.

The required adjustments in the input combination condition depend on the selection of the monitoring methods. If the regulator decides monitoring only outputs and the monitoring cost will not be influenced by the emission level, \( \partial M(n^*, q^*, e^*)/\partial e = 0 \). Then, the marginal factor cost to society will be reduced to \( r_j + D'(n^*e^*) \cdot \partial h(x^*)/\partial x_j, j = 1, \ldots, m \). Alternatively, if the regulator decides monitoring only emissions and the monitoring cost will not be influenced by the output level, \( \partial M(n^*, q^*, e^*)/\partial q = 0 \). Then, the marginal factor benefit to society will be the marginal revenue product of each input, \( P(n^*q^*) \cdot \partial f(x^*)/\partial x_j, j = 1, \ldots, m \).

The nature of the pollution generations and the availability of monitoring methods determine the selection of the monitoring method. Although the selection of monitoring methods is an interesting and important research agenda, we want to keep it beyond the scope of this present discussion. Rather than discussing the selection of monitoring method, we will simply assume that monitoring activities are mandatory and incur some costs.

Consider the entry condition of (24). If the monitoring cost does not depend on the number of firms in the industry, \( \partial M(n, q, e)/\partial n = 0 \), the entry condition of (24) becomes identical with (5). Further, if the number of firms is large enough, the input combination conditions of (25) will be reduced to (6). It means that the effect of the firm’s production choices have only trivial effect on the monitoring cost.

If the monitoring cost increases in proportion to the total output level \( Q \) and total input level \( E \), the monitoring cost will be \( M(n, q, e) = M(nq, ne) = M(Q, E) \). The input combination conditions of (25) will be

\[
\left( P(n^*q^*) - \frac{\partial M(Q^*, E^*)}{\partial Q} \right) \frac{\partial f(x^*)}{\partial x_j} = r_j + \left( D'(n^*e^*) + \frac{\partial M(Q^*, E^*)}{\partial E} \right) \frac{\partial h(x^*)}{\partial x_j}
\]

\[ j = 1, \ldots, m. \]

The number of firms in the industry will not influence the optimal input combination.

When the growth rate of the monitoring cost is an increasing function of the number of firms, the regulator has to do cumbersome works. In particular, the regulator has to vary tax rates with the number of firms in the industry (see equation (25)). When the number of firms is large, the
increase of the monitoring cost through the output expansion should be relatively small. Hence, a relatively small adjustment is enough. On the contrary, when the number of firms is small, the increase of the monitoring cost through the output expansion would be large. Hence, a relatively large adjustment is required. The regulator has to do a similar kind of job for the adjustment of the second square bracket in equation (25).

5. Regulation Under Imperfect Monitoring

To discuss the validity of tax schemes, we have to assume how firms behave under imperfect monitoring. For conclusive analysis of these topics, we need to adopt principal-agent approaches. However, applications of these approaches will be beyond the scope of this paper. In this paper, we adopt the Harford model and simply assume that the firm will behave as if there was no emission tax without any monitoring cost (see Harford 1978 and Schmutzler and Goulder 1997). Denote actual emissions $e$ and undeclared emissions $e_u$ where $e_u \in [0,e]$, respectively. The firm’s problem is to choose output, emission, undeclared emission, and inputs so as to maximize the (expected) profit subject to technological constraints. The Lagrangian for the firm is

$$
\ell = Pq - \sum_{j=1}^{m} r_j x_j - F - t_e (e - e_u) - z(e_u, M) - \lambda(f(X) - q) + \sigma (e - h(X)) \tag{26}
$$

where $z(e_u, M)$ is the expected penalty fine given the monitoring expense $M$. When the regulator spends a large amount of money for the monitoring activities, the firm faces a higher probability of paying a penalty fine. Hence, the larger the monitoring expense, the smaller undeclared emissions will be. The first-order necessary conditions include

$$
\frac{\partial \ell}{\partial q} : P - \lambda = 0 \tag{27}
$$

$$
\frac{\partial \ell}{\partial e} : - t_e + \sigma = 0 \tag{28}
$$

$$
\frac{\partial \ell}{\partial e_u} : t_e - \frac{\partial z(e_u, M)}{\partial e_u} = 0 \tag{29}
$$
The relationship (29) tells us the firm chooses the level of undeclared emissions so that the expected marginal penalty fine equals the emission tax. We assume that the regulator applies such a fine system with the monitoring cost $M^2$.

Combining the first-order conditions, we have the input combination condition,

$$
\frac{\partial l}{\partial x_j} = -r_j + \lambda \frac{\partial f(X)}{\partial x_j} - \sigma \frac{\partial h(X)}{\partial x_j} = 0 \quad j = 1, \ldots, m.
$$

(30)

By comparing equation (31) with equation (25), we can discuss the limitations of Pigouvian taxes as a remedy for externalities. First, the input combination conditions will not be achieved with a pure emission tax. It is because the marginal factor benefit will not be adjusted to the socially optimal level by emission taxes. As long as the monitoring cost depends on the output level, we require some adjustment mechanisms so as to subtract the increase of the monitoring cost, $(1/n^*)\frac{\partial M(n^*, q^*, e^*)}{\partial q}$, from the output price, $P(n^* q^*)$.

An output tax can be used for this purpose. Defining $t_o$ as an output tax and solving the firm’s maximization problem, we will have the input combination condition,

$$
P \frac{\partial f(X)}{\partial x_j} = r_j + t_e \frac{\partial h(X)}{\partial x_j} \quad j = 1, \ldots, m.
$$

(31)

Setting the output tax $t_o = (1/n^*)\frac{\partial M(n^*, q^*, e^*)}{\partial q}$ and the emission tax $t_e = D'(n^* e^*) + (1/n^*)\frac{\partial M(n^*, q^*, e^*)}{\partial e}$, the input combination condition of (25) will be achieved.

**PROPOSITION 4.** If a monitoring cost depends on the output level, a pure emission tax will not achieve the input combination condition. The input combination condition will be achieved with a mixed tax.

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2. When the equality is not hold, we will have corner solutions. If $t, < \partial z(e, M)/\partial e$, the firm never pays emission taxes. Paying penalty fees is a cheaper option for the firm. Hence, $e = e^*$ and the emission tax revenue is zero. If $t, > \partial z(e, M)/\partial e$, the firm declares the full amount of emissions. Paying emission taxes is a cheaper option for the firm. Hence, $e = 0$ and the penalty fine is zero.
To provide correct entry incentives to the firms, the long-run entry condition (24) needs to be satisfied. For examining the long-run entry condition in a simple framework, we assume that the monitoring cost depends on total output and total emission. Under this condition, \( M(n, q, e) = M(nq, ne) = M(Q, E) \). Then, the optimal output tax is \( t^*_o = \left( \frac{\partial M(Q^*, E^*)}{\partial Q} \right) \) and the optimal emission tax is \( t^*_e = \left( \frac{\partial M(Q^*, E^*)}{\partial E} \right) \), respectively. The sum of total tax bill and penalty fine paid by individual firms is \( t_o q + t_e (e - e_u) + z(e_u, M) \). On the other hand, the necessary adjustment is \( t_o q + t_e e \) (see equation (24)). This is the amount of money to be subtracted from the long-run zero profit condition. Clearly, the firms incur appropriate costs when \( t_e e_u = z(e_u, M^*) \). It requires that the emission tax coincides with the (expected) penalty fine and \( \frac{\partial z(e_u, M^*)}{\partial e_u} = \frac{z(e_u, M^*)}{e_u} \) from equation (29). When this condition is satisfied, it does not matter whether the firms honestly reports their emissions or not. Anyhow, the same amount of money will be collected as penalty fines. To hold this equality for all range of \( e_u \), an expected penalty fine of \( z(e_u, M) \) has to be a linear function of \( e_u \).

When the above equality is not satisfied, a lump-sum tax (or subsidy),

\[
T = \left[ t_e - \frac{z(e_u, M^*)}{e_u} \right] e_u^* = \left[ \frac{\partial z(e_u, M^*)}{\partial e_u} - \frac{z(e_u, M^*)}{e_u} \right] e_u^*,
\]

is necessary to achieve the optimal entry condition. It is because the firm actually pays \( z(e_u, M^*) \) although it needs to pay \( \frac{\partial z(e_u, M^*)}{\partial e_u} + \frac{z(e_u, M^*)}{e_u} \) as a penalty fine.

**PROPOSITION 5.** If a firm can choose the level of undeclared emissions, the entry condition will not be achieved with pigouvian taxes. A transfer payment is required in order to adjust an expected penalty fine to the social damage of undeclared emissions.

Suppose a firm has to pay a penalty fine of \( \alpha e_u \) when the regulator detects he or she is cheating. \( \alpha \) is a penalty fine per emission. If a probability detected by the regulator is a function of the level of undeclared emissions and the monitoring expense, an expected penalty fine is \( z(e_u, M) = \pi(e_u, M)\alpha e_u \). When input combination conditions are satisfied, the expected penalty fine paid by an individual firm is \( z(e_u^*, M^*) = \pi(e_u^*, M)\alpha e_u^* \). On the contrary, the actual social damage of undeclared emissions is \( \frac{\partial z(e_u^*, M)}{\partial e_u} e_u^* = \partial \pi(e_u^*, M)/\partial e_u a(e_u)^2 + \pi(e_u^*, M)\alpha e_u^* \). Clearly, the expected penalty fine
is smaller than the social damage of undeclared emissions and a lump-sum tax of \( T = \frac{\partial \pi (e^{*}, M^*)}{\partial e_n} a(e^{*}) \) is necessary to achieve the appropriate entry condition.

An implication of this result is as follows. By choosing pigouvian taxes to the short-run optimal level, the regulator can achieve the input combination condition. Then, he or she will succeed to induce the firm to choose the social optimal output and emission. However, it does not necessarily mean the regulator simultaneously achieves the long-run condition. Even if pigouvian taxes are appropriately imposed, the firms are able to avoid paying their social costs by under-reporting their emissions. To achieve the entry condition, a lump-sum tax is necessary.

6. Conclusion and Policy Implication

In this paper, we discussed emission regulation and long-run optimality. To derive long-run optimality, the regulator has to achieve two conditions. Those are the input combination condition and the entry condition. Based on these two conditions, we evaluated various emission regulations. Some of the results in this paper might be useful for the design of emission regulations.

First, input taxes achieve neither the input combination condition nor the entry condition. Unless imposing input taxes on all inputs, the social optimal condition will not be obtained. Imposing input taxes on selected inputs create distortions in the allocation of resources. Further, for the application of input taxes, a transfer payment is necessary. Without a transfer payment, the regulator cannot provide correct entry incentives to the firms.

Second, under imperfect monitoring, a pure emission tax unlikely obtains the social optimality conditions. A monitoring cost will increase as the output level increases. This monitoring cost has to be subtracted from the marginal social benefit of output. However, the regulator cannot do this job alone with emission taxes. An introduction of an output tax can solve this problem.

Third, a transfer payment is necessary to achieve the long-run entry condition under imperfect monitoring. Even if the regulator succeeds to choose the optimal tax rates, the long-run entry condition may not be obtained. Only when the expected penalty fine equals the actual social
damage of undeclared emissions, the long-run entry condition is satisfied. Otherwise, the regulator has to use a transfer payment.

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