Optimal Banking Contracts with an Informed Bank

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Abstract

This paper concerns optimal banking contracts in the case of common value, that is, where the bank's utility as well as the firm's profit functions depend on the state of nature. We assume that an informed bank is strictly risk averse and makes a contractual offer to a firm who is risk neutral. According to Laffont and Martimort (2002, chapter 9), to avoid the difficult issues of signaling, we assume that the bank makes his contractual offer before he learns the state of nature θ. This paper has pointed out that the firm's wealth constraint as well as risk preferences of both parties play an important role in financial contracting. When the firm is wealth constrained, an upward distortion for the lending is obtained in efficient state θ_1. On the contrary, when the firm is not wealth constrained, a downward distortion for the lending is obtained in inefficient state θ_0. Different conclusions are reached, though, if the bank as well as the firm is risk neutral. That is, no allocative inefficiency is obtained in the case of the wealth-unconstrained firm, while allocative inefficiency is still obtained in the case of the wealth-constrained firm.

Keywords: Common value, informed principal, wealth constraint, risk preference, banking contracts.

1 Introduction

This paper concerns optimal banking contracts designed by the informed bank. Two problems of hidden information have been distinguished. One is the screening problem[1] and the other the signaling problem.[2] In the context of financial contracting the first problem refers to a situation where the uninformed bank makes a contractual offer to the informed firm. That is, the information about the profitability of the firm's investment is hidden from the bank. The second problem refers to the opposite situation where the informed bank makes a contractual offer to the uninformed firm. That is, the information about the efficiency of the bank's lending is hidden from the firm.

[1]The screening problem was first formally analyzed by Mirrlees (1971). For the literature on this problem see Bolton and Dewatripont (2005, chapter 2), Laffont and Martimort (2002, chapters 2 and 3), and Salanié (2005, chapters 2 and 3).

[2]The classic example of a signaling problem is the model of education as a signal by Spence (1973, 1974). For the literature on this problem see Bolton and Dewatripont (2005, chapter 3), Laffont and Martimort (2002, chapter 9), and Salanié (2005, chapter 4).

We focus on optimal banking contracts in the case of common value, that is, where the bank's utility as well as the firm's profit functions depend on the state of nature. We assume that an informed bank is strictly risk averse and makes a contractual offer to a firm who is risk neutral.[3] According to Laffont and Martimort (2002, chapter 9), to avoid the difficult issues of signaling, we assume that the bank makes his contractual offer before he learns the state of nature θ. The timing of this *ex ante* contracting is as follows:

- $t = 0$: The bank offers a contract.
- $t = 1$: The firm accepts or refuses the contract.
- $t = 2$: The bank discovers his type θ.
- $t = 3$: The contract is executed.

As the theory of incomplete contracts mentions, the firm's wealth constraint plays an important part in financial contracting.[4] In this paper as well as in Laffont and Martimort (2002, chapter 9), we analyze two cases: one where the firm is wealth constrained and the firm's *ex post* participation constraints are assumed; the other where the firm is not wealth constrained and the firm's *ex ante* participation constraint is assumed.[5]

The structure of this paper is as follows. Section 2 gives a fundamental setup of the model. Section 3 presents the optimal first-best contracts as a benchmark for the analysis. Section 4 analyzes the case with the wealth-constrained firm under asymmetric information. It is shown for this case that an upward distortion for the lending is obtained in efficient state. Section 5 examines the case with the wealth-unconstrained firm under asymmetric information. It is shown for this case that a downward distortion for the lending is obtained in inefficient state. Different conclusions are reached, though, if the bank as well as the firm is risk neutral. The systematic differences are also explained in Section 5. Finally, Section 6 summarizes the main findings.

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[3]On the contrary, in Laffont and Martimort (2002, chapter 9) it has been assumed that the informed principal is risk neutral and that the agent is strictly risk averse. In this paper we consider a case where in financial contracting a lender is relatively risk averse when compared with a borrower.

[4]Financial contracting has been a fruitful application of incomplete contract theory, following the contributions of Aghion and Bolton (1992), Bolton and Scharfstein (1990), and Hart and Moore (1989, 1994, 1998). See for example Bolton and Dewatripont (2005, part IV) for incomplete contract theory.

[5]For *ex post* and *ex ante* (or *interim*) participation constraints see Crémer and McLean (1985, 1988), McAfee and Reny (1992), and Neeman (2004).

2 Setup

The informed principal is a lender (or a bank) who provides a loan of size l to a borrower (or a firm). The cost of providing a loan of size l is $C(l, \theta)$ for which we assume that $C(0, \theta) = 0, C_l(l, \theta) > 0$ for $l > 0, C_l(0, \theta) = 0, C_{ll}(l, \theta) > 0$ for $l \geq 0$, and the Spence-Mirrlees property $C_{l\theta}(l, \theta) < 0$ for $l > 0$ are satisfied. The firm's repayment to the bank is r. Since the bank makes a profit $r - l - C(l, \theta)$, he has thus a utility function $V = v(r - c(l, \theta))$ where $c(l, \theta) \equiv l + C(l, \theta)$ is the gross cost of lending and $v(\cdot)$ is some increasing and strictly concave utility function ($v' > 0, v'' < 0$ with $v(0) = 0$). The firm makes a profit $U = \theta l - r$ where θl is the gross return of investment with l units of loan. We normalize the both parties' outside opportunity levels of utility to 0. The parameter θ is the marginal (gross) return of investment drawn from $\Theta = \{\theta_0, \theta_1\}$ with respective probabilities p and $1 - p$. A party with θ_0 is an inefficient type and a party with θ_1 an efficient type. We assume that $\theta > 1$, that is, the gross return of investment with l units of loan must always be strictly greater than the units of loan, $\theta l > l$. The Spence-Mirrlees property $C_{l\theta}(l, \theta) < 0$ means that the higher θ leads to the lower marginal cost of lending.

According to the revelation principle, there is no loss of generality in restricting the bank to offer direct revelation mechanisms of the kind $\{(l_0, r_0); (l_1, r_1)\}$. For further references we let $R_0 \equiv r_0 - c(l_0, \theta_0)$ and $R_1 \equiv r_1 - c(l_1, \theta_1)$ denote the bank's information rents in both states of nature. For notational simplicity let $V_0 \equiv v(R_0)$ and $V_1 \equiv v(R_1)$. We can replace the menu of contracts $\{(l_0, r_0); (l_1, r_1)\}$ by the menu of contracts $\{(l_0, R_0); (l_1, R_1)\}$ to perform the optimization of the bank's problem.

The bank being informed of his type *ex post*, any contract that he offers at the *ex ante* stage must satisfy the following incentive compatibility constraints of the bank:

$$R_0 \geq R_1 - \Phi(l_1), \tag{ic$_0$}$$

$$R_1 \geq R_0 + \Phi(l_0), \tag{ic$_1$}$$

where $\Phi(l) = C(l, \theta_0) - C(l, \theta_1)$. Because of the assumptions made on $C(\cdot)$, $\Phi(l)$ increases with units of loan $\Phi' > 0$ for $l > 0$, and satisfies $\Phi(0) = 0, \Phi'(0) = 0$. Summing these two incentive compatibility constraints and using that $\Phi' \geq 0$, we obtain the monotonicity condition

$$l_1 \geq l_0. \tag{1}$$

We assume that the firm is wealth constrained and does not have the financial resources to meet the repayments more than the gross returns of investment. Therefore, the firm's participation constraint is written as a pair of *ex post* participation constraints, one for each state of nature:

$$\theta_0 l_0 - c(l_0, \theta_0) - R_0 \geq 0, \tag{pc$_0$}$$

$$\theta_1 l_1 - c(l_1, \theta_1) - R_1 \geq 0. \tag{pc$_1$}$$

In section 5, we will consider the case where the firm is not wealth constrained.

In what follows, we can neglect the bank's *ex ante* participation constraint and assume that the bank makes a take-it-or-leave-it offer to the firm, because the bank has all the bargaining power at the *ex ante* stage when the contract is offered. The bank's optimization problem is

Problem I

$$\max_{\{(l_0, R_0);\,(l_1, R_1)\}} pv(R_0) + (1-p)v(R_1) \tag{2}$$

$$\text{subject to } (\text{ic}_0),\ (\text{ic}_1),\ (\text{pc}_0), \text{ and } (\text{pc}_1).$$

Indeed, the bank must maximize his *ex ante* payoff subject to the firm's participation constraints and to his own incentive compatibility constraints, ensuring that *ex post*, that is, once he has learned the state of nature, he will truthfully reveal this state of nature.

3 Symmetric Information

As a benchmark, let us consider the case of symmetric information. Under symmetric information — that is, knowing θ — the bank would solve

$$\max_{\{l_i, r_i\}} r_i - c(l_i, \theta_i) \tag{3}$$

subject to $U_i = \theta_i l_i - r_i \geq 0$. The solution of this problem is

$$\theta_i = c_l(l_i, \theta_i) \quad \text{or} \quad l_i \equiv l_i^{fb} > 0, \tag{4}$$

$$U_i = \theta_i l_i^{fb} - r_i = 0 \quad \text{or} \quad r_i^{fb} \equiv \theta_i l_i^{fb}. \tag{5}$$

That is, the marginal return of investment, θ_i, must be equal to the marginal cost of lending, $c_l(l_i, \theta_i)$. The firm receives no rent, because the bank has all the bargaining power at the *ex ante* stage. The Spence-Mirrlees property $C_{l\theta} = c_{l\theta} < 0$ ensures that the monotonicity condition always holds for the first-best loan, that is, $l_0^{fb} < l_1^{fb}$.

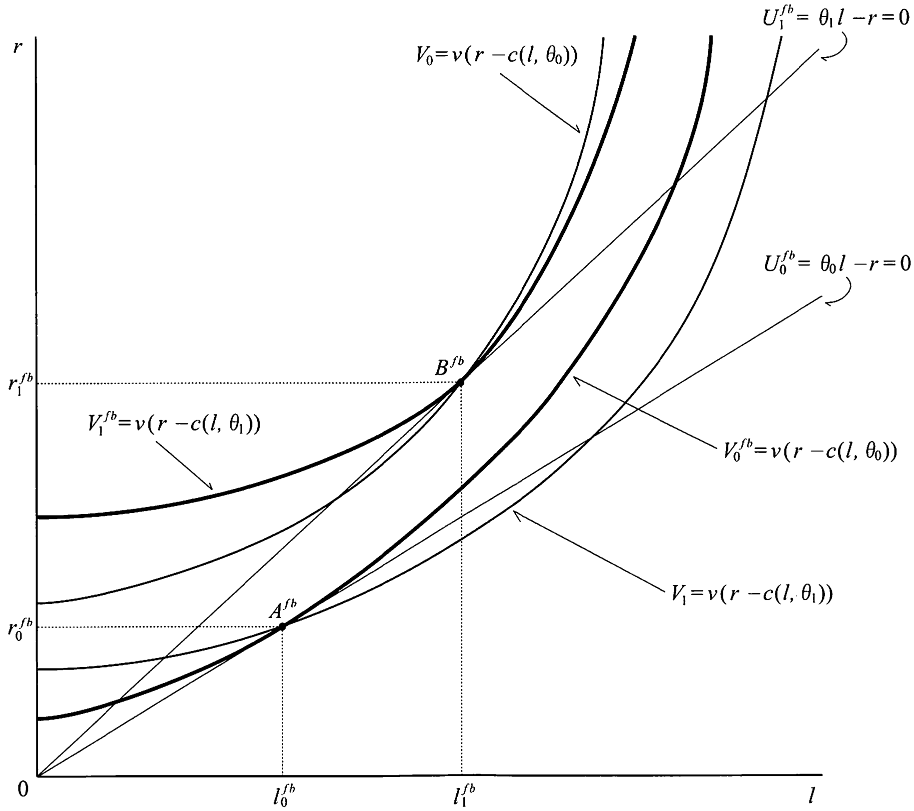

Figure 1: First-Best Contracts with an Informed Bank

According to Laffont and Martimort (2002, chapter 9), in order to make the problem interesting, we assume that the incentive compatibility constraint (ic_0) is not satisfied by the first-best allocation. Using (5), this occurs if

$$\begin{aligned} R_0^{fb} - R_1^{fb} &= \theta_0 l_0^{fb} - c(l_0^{fb}, \theta_0) - [\theta_1 l_1^{fb} - c(l_1^{fb}, \theta_1)] \\ &< -c(l_1^{fb}, \theta_0) + c(l_1^{fb}, \theta_1) = -\Phi(l_1^{fb}), \end{aligned}$$

which holds if

$$\theta_0 l_0^{fb} - c(l_0^{fb}, \theta_0) < \theta_1 l_1^{fb} - c(l_1^{fb}, \theta_0). \tag{6}$$

We have presented the optimal first-best contracts A^{fb} and B^{fb}, offered in the states of nature θ_0 and θ_1, respectively, in figure 1. Note that the Spence-Mirrlees property $C_{l\theta} = c_{l\theta} < 0$ ensures that the indifference curve of the θ_0-type of the bank crosses the indifference curve of the θ_1-type of the bank only once and then has a steeper slope, as in figure 1. A higher level of the bank's utility is obtained when the isoutility curve moves in the northwest direction.

4 Asymmetric Information and a Wealth-Constrained Firm

Let us move now to the case of asymmetric information, where only the bank knows the value of θ. By moving from A^{fb} to B^{fb} when state θ_0 realizes, the bank can increase his expected profit. On the contrary, the bank never wants to offer A^{fb} when he should offer B^{fb} in state θ_1.

The previous analysis suggests that (ic_0) is the relevant incentive compatibility constraint in problem I when (6) holds. To maximize expected payoff (2) under (ic_0), (ic_1), (pc_0), and (pc_1), we momentarily neglect (ic_1), and we later check that the solution of the maximization under (ic_0), (pc_0), and (pc_1) satisfies (ic_1). Denoting the multipliers of (ic_0), (pc_0), and (pc_1), by γ, λ_0, and λ_1, respectively, and optimizing with respect to l_0, l_1, R_0, and R_1 yields

$$\lambda_0[\theta_0 - c_l(l_0^*, \theta_0)] = 0, \tag{7}$$

$$\lambda_1[\theta_1 - c_l(l_1^*, \theta_1)] + \gamma\Phi'(l_1^*) = 0, \tag{8}$$

$$pv'(R_0^*) - \lambda_0 + \gamma = 0, \tag{9}$$

$$(1-p)v'(R_1^*) - \lambda_1 - \gamma = 0, \tag{10}$$

where $\{(l_0^*, R_0^*);\ (l_1^*, R_1^*)\}$ is the second-best allocation.

If $\gamma = 0$ held, (9) and (10) would, respectively, suggest that $\lambda_0 > 0$ and $\lambda_1 > 0$, which, together with $\gamma = 0$, would yield the first-best allocation by (7) and (8). However, according to the assumption (6) the incentive compatibility constraint (ic_0) is not satisfied by the first-best allocation. Hence, $\gamma > 0$. As expected, the bank's incentive compatibility constraint (ic_0) is binding. (9) implies that $\lambda_0 > 0$ which combines with (7) to yield $\theta_0 = c_l(l_0^*, \theta_0)$ or $l_0^* = l_0^{fb}$. Because the monotonicity condition (1) implies that $l_1^* \geq l_0^*$, we get $l_1^* > 0$. (8) and $l_1^* > 0$ imply that $\lambda_1[\theta_1 - c_l(l_1^*, \theta_1)] = -\gamma\Phi'(l_1^*) < 0$ to yield $\lambda_1 > 0$ and $\theta_1 < c_l(l_1^*, \theta_1)$ which implies that $l_1^* > l_1^{fb}$ since $c(\cdot)$ is strictly convex. Consequently, we have

$$\begin{gathered}\gamma > 0, \lambda_0 > 0, \lambda_1 > 0, \text{and} \\ l_0^* = l_0^{fb} < l_1^{fb} < l_1^*.\end{gathered}$$

Note that using $c_l(l_1^*, \theta_1) > \theta_1$, the Spence-Mirrlees property $C_{l\theta} = c_{l\theta} < 0$ implies that

$$c_l(l_1^*, \theta_0) > c_l(l_1^*, \theta_1) > \theta_1.$$

Using (8), (9), and (10), we obtain

$$\gamma = \frac{(1-p)v'(R_1^*)[\theta_1 - c_l(l_1^*, \theta_1)]}{\theta_1 - c_l(l_1^*, \theta_0)}, \tag{11}$$

$$\lambda_0 = pv'(R_0^*) + \frac{(1-p)v'(R_1^*)[\theta_1 - c_l(l_1^*, \theta_1)]}{\theta_1 - c_l(l_1^*, \theta_0)}, \tag{12}$$

$$\lambda_1 = -\frac{(1-p)v'(R_1^*)\Phi'(l_1^*)}{\theta_1 - c_l(l_1^*, \theta_0)}. \tag{13}$$

Because $\gamma > 0$, $\lambda_0 > 0$, and $\lambda_1 > 0$ imply that the constraints (ic_0), (pc_0), and (pc_1) are binding at the optimum, we have

$$R_1^* = R_0^* + \Phi(l_1^*), \tag{ic_0'}$$

$$R_0^* = \theta_0 l_0^{fb} - c(l_0^{fb}, \theta_0), \tag{pc_0'}$$

$$R_1^* = \theta_1 l_1^* - c(l_1^*, \theta_1). \tag{pc_1'}$$

R_0^* is straightforwardly given by (pc_0'), and l_1^* is determined by

$$\theta_1 l_1^* - c(l_1^*, \theta_0) = \theta_0 l_0^{fb} - c(l_0^{fb}, \theta_0).$$

Finally R_1^* is determined by (pc_1'). (ic_0') and $l_1^* > 0$ imply that $R_1^* > R_0^*$.

Note that the neglected constraint (ic_1) is satisfied by this solution. Using (ic_0'), (ic_1) can be rewritten as

$$0 \geq \Phi(l_0^{fb}) - \Phi(l_1^*), \tag{14}$$

which is true since $l_0^{fb} < l_1^*$ and $\Phi' > 0$ for $l > 0$.

We can summarize our findings in proposition 1.

Proposition 1: Assume that the firm is risk neutral and is wealth constrained and that the informed bank is strictly risk averse and makes the contractual offer at the *ex ante* stage. Then, the optimal contract entails the following:

1. Both the firm's *ex post* participation constraints in both states and the bank's incentive compatibility constraint in state θ_0 are binding.

2. No loan distortion for the lending that is obtained when θ_0 realizes $l_0^* = l_0^{fb}$.

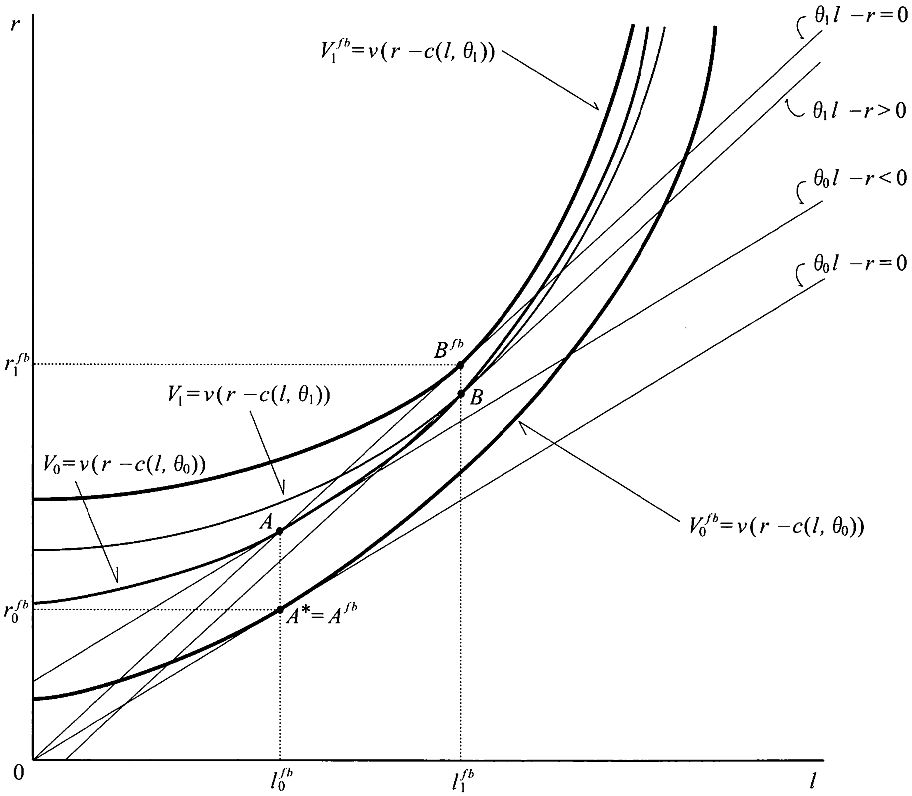

Figure 2: Incentive Compatible Contracts with a Wealth-Constrained Firm

3. An upward distortion for the lending that is obtained when θ_1 realizes $l_1^* > l_1^{fb}$, where

$$\theta_1 l_1^* - c(l_1^*, \theta_0) = \theta_0 l_0^{fb} - c(l_0^{fb}, \theta_0). \tag{15}$$

To understand the results of proposition 1, note that the bank's incentive compatibility constraint (ic_0) in state θ_0 is more easily satisfied when R_0 increases, R_1 decreases and l_1 increases with respect to the symmetric information optimal contract. Since only the incentive compatibility constraint (ic_0) is binding, there is no need to distort the lending when state θ_0 realizes. Under asymmetric information as well as under complete information, the firm's payoff is zero in each state of nature.

These results are represented graphically in figures 2 and 3. Keeping the same loans as under symmetric information but increasing (respectively decreasing) the bank's payoff when θ_0 (respectively θ_1) realizes, could the bank offer the incentive compatible menu of

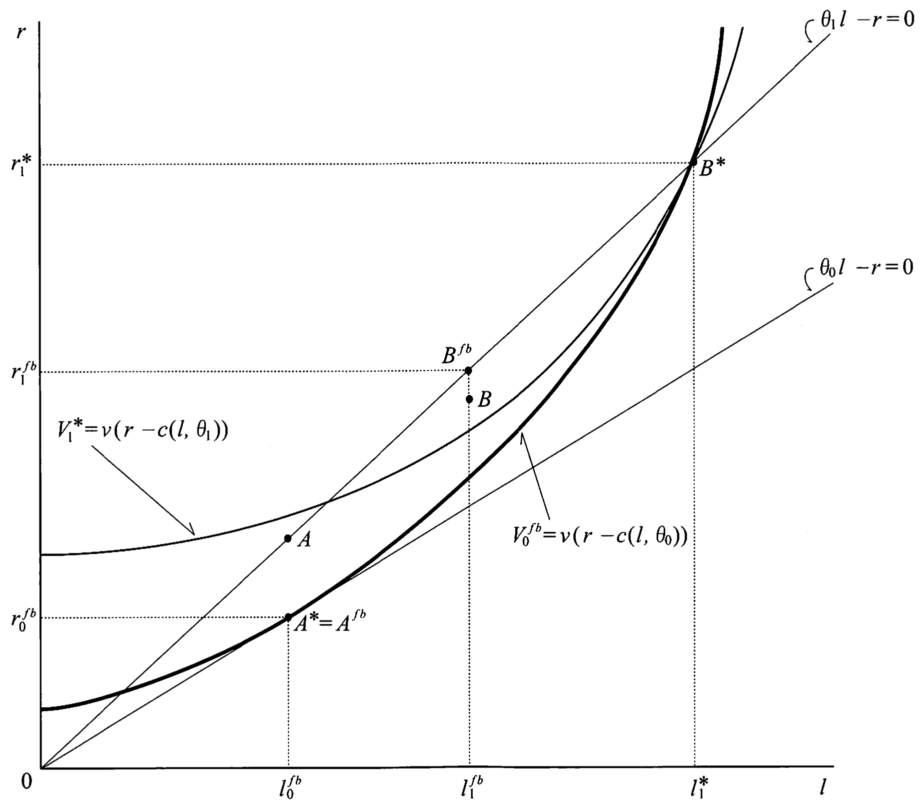

Figure 3: Second-best Contracts with a Wealth-Constrained Firm

contracts (A, B) to the wealth-constrained firm?

This menu is incentive compatible because the bank is indifferent between contracts A and B in state θ_0 and strictly prefers B to A in state θ_1. However, this menu is not acceptable to the firm, because the firm's *ex post* participation constraint (pc_0) does not hold when θ_0 realizes. Slightly decreasing r_0, that is, moving from A to A^*, while moving from B to B^* on the indifferent curve of the bank in state θ_0, which goes through A^*, increases (respectively decreases) the firm's *ex post* payoff to zero when θ_0 (respectively θ_1) realizes. Doing so creates an efficiency loss, because l_1^{fb} is maximizing allocative efficiency. This distortion is finally optimal for the pair of contracts (A^*, B^*). Moreover, l_1^* is the loan closest to l_1^{fb} on the zero-profit line of the firm in state θ_1, such that the bank's incentive compatibility constraint in state θ_0 remains satisfied.

5 Asymmetric Information and a Wealth-Unconstrained Firm

Let us now assume that the firm is not wealth constrained. The *ex post* participation constraints (pc_0) and (pc_1) can be replaced by the *ex ante* participation constraint:

$$p[\theta_0 l_0 - c(l_0, \theta_0) - R_0] + (1-p)[\theta_1 l_1 - c(l_1, \theta_1) - R_1] \geq 0. \tag{pc}$$

Let us consider two cases in turn: one where the bank is strictly risk averse; the other where the bank is risk neutral.

5.1 The Case of a Strictly Risk-Averse Bank

The bank's optimization problem is now

Problem II

$$\max_{\{(l_0,R_0);\,(l_1,R_1)\}} pv(R_0) + (1-p)v(R_1) \tag{16}$$

subject to (ic_0), (ic_1), and (pc).

To maximize expected payoff (16) under (ic_0), (ic_1), and (pc), we momentarily neglect (ic_0), and we later check that the solution of the maximization under (ic_1) and (pc) satisfies (ic_0). Denoting the multipliers of (ic_1) and (pc), by λ and μ, respectively, and optimizing with respect to l_0, l_1, R_0, and R_1 yields

$$-\lambda\Phi'(l_0^*) + \mu p[\theta_0 - c_l(l_0^*, \theta_0)] = 0, \tag{17}$$

$$\mu(1-p)[\theta_1 - c_l(l_1^*, \theta_1)] = 0, \tag{18}$$

$$pv'(R_0^*) - \lambda - \mu p = 0, \tag{19}$$

$$(1-p)v'(R_1^*) + \lambda - \mu(1-p) = 0. \tag{20}$$

Summing (19) and (20), we obtain

$$\mu = pv'(R_0^*) + (1-p)v'(R_1^*) > 0. \tag{21}$$

Hence (pc) is necessarily binding at the optimum.

Substituting (21) into (19), we get

$$\lambda = p(1-p)[v'(R_0^*) - v'(R_1^*)]. \tag{22}$$

Because $v(\cdot)$ is strictly concave, λ is positive if and only if $R_0^* < R_1^*$.[6]

Since $\mu > 0$, $\lambda > 0$, and $\Phi'(l_0^*) > 0$, (17) and (18), respectively, implies

$$\theta_0 - \frac{\lambda}{p\mu}\Phi'(l_0^*) = c_l(l_0^*, \theta_0), \tag{23}$$

and

$$\theta_1 = c_l(l_1^*, \theta_1). \tag{24}$$

Because $c(\cdot)$ is strictly convex, $\Phi'(l_0^*) > 0$ and $l_0^{fb} < l_1^{fb}$, we obtain

$$l_0^* < l_0^{fb} < l_1^{fb} = l_1^*. \tag{25}$$

Note that the neglected constraint (ic_0) is satisfied by this solution. Because (ic_1) is binding at the optimum, (ic_0) can be rewritten as

$$0 \geq \Phi(l_0^*) - \Phi(l_1^{fb}), \tag{26}$$

which is true since $l_0^* < l_1^{fb}$ from (25) and $\Phi' > 0$ for $l > 0$.

Lastly, it is easy to check that the firm gets a negative payoff when θ_0 occurs and a positive payoff when θ_1 realizes instead.[7]

We can summarize our findings in proposition 2.

Proposition 2: Assume that the firm is risk neutral and is not wealth constrained and that the informed bank is strictly risk averse and makes the contractual offer at the *ex ante* stage. Then, the optimal contract entails the following:

1. Both the firm's *ex ante* participation constraint and the bank's incentive compatibility constraint in state θ_1 are binding.

2. No loan distortion for the lending that is obtained when θ_1 realizes $l_1^* = l_1^{fb}$.

[6] Let us check that $R_0^* < R_1^*$. If $l_0^* = 0$, (17), $\Phi'(0) = 0$, and $\mu > 0$ would imply that $\theta_0 = c_l(l_0^*, \theta_0)$ or $l_0^* = l_0^{fb} > 0$. Hence, we have $l_0^* > 0$ and are not in the case of a contract with shutdown. If $R_0^* \geq R_1^*$, (ic_1) would imply that $0 \geq R_1^* - R_0^* \geq \Phi(l_0^*)$ which conflicts with $l_0^* > 0$ because of the assumptions made on $\Phi(\cdot)$. Therefore, we have $R_0^* < R_1^*$.

[7] Using the expression obtained when (ic_1) is binding, we obtain

$$\begin{aligned}\theta_1 l_1^* - r_1^* - (\theta_0 l_0^* - r_0^*) &= \theta_1 l_1^{fb} - \theta_0 l_0^* - c(l_1^{fb}, \theta_1) + c(l_0^*, \theta_1) \\ &= \int_0^{l_0^*} (\theta_1 - \theta_0) dl + \int_{l_0^*}^{l_1^{fb}} [\theta_1 - c_l(l, \theta_1)] dl > 0,\end{aligned}$$

which is true since $\theta_1 > \theta_0$ and $\theta_1 > c_l(l, \theta_1)$ for all $l \in [0, l_1^{fb})$. Because (pc) is binding, we have

$$\theta_0 l_0^* - r_0^* < 0 < \theta_1 l_1^* - r_1^*.$$

3. A downward distortion for the lending that is obtained when θ_0 realizes $l_0^* < l_0^{fb}$, where

$$\theta_0 - \frac{(1-p)[v'(R_0^*) - v'(R_1^*)]\Phi'(l_0^*)}{pv'(R_0^*) + (1-p)v'(R_1^*)} = c_l(l_0^*, \theta_0). \tag{27}$$

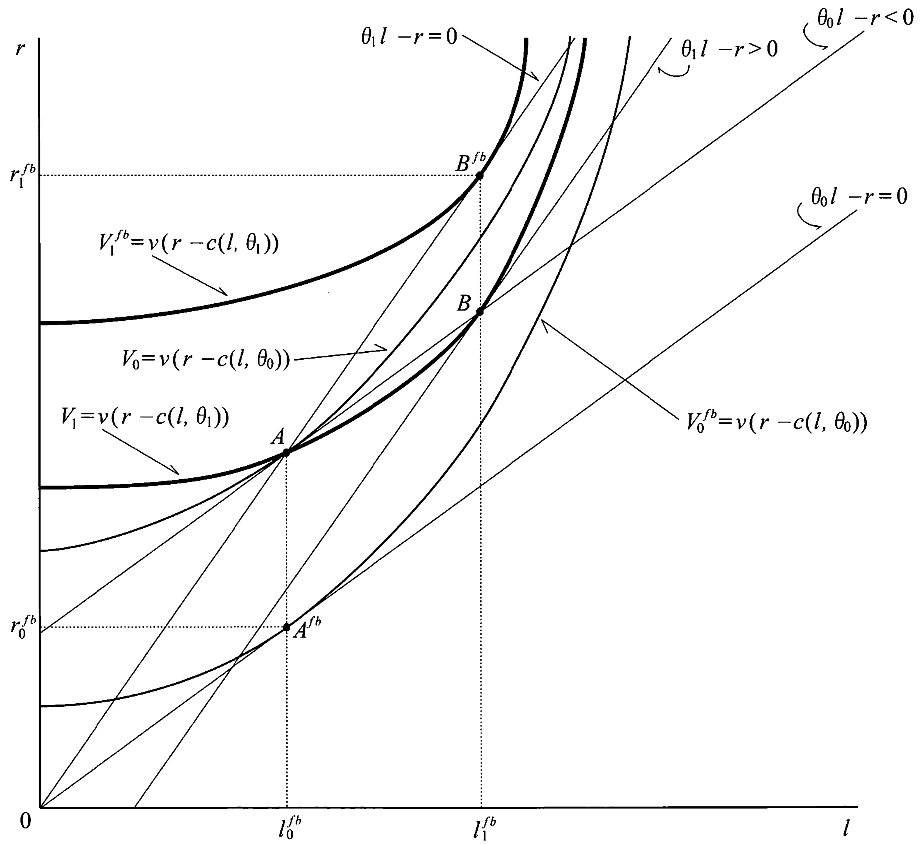

Figure 4: Incentive Compatible Contracts with a Wealth-Unconstrained Firm

These results are represented graphically in figures 4 and 5. Keeping the same loans as under symmetric information but increasing (respectively decreasing) the bank's payoff when θ_0 (respectively θ_1) realizes, the bank could offer the incentive compatible menu of contracts (A, B).

This menu is incentive compatible because the bank is indifferent between contracts A and B in state θ_1 and strictly prefers A to B in state θ_0. However, this menu imposes too much risk on the bank. Slightly decreasing r_1, that is, moving from B to B^*, while moving from A to A^* on the indifference curve of the bank in state θ_1, which goes through B^*, decreases risk and still preserves incentive compatibility (see figure 5). Doing so creates an efficiency loss, because l_0^{fb} is maximizing allocative efficiency. This distortion is finally optimal for the pair of contracts (A^*, B^*).

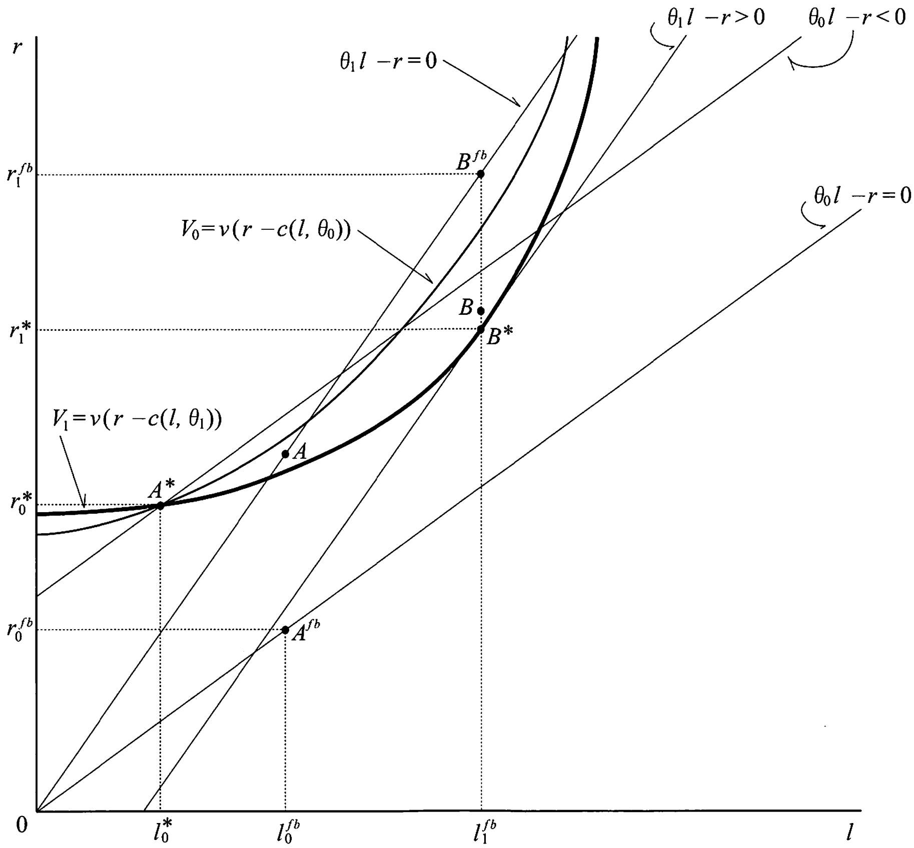

Figure 5: Second-best Contracts with a Wealth-Unconstrained Firm

5.2 The Case of a Risk-Neutral Bank

Let us assume that the bank is risk neutral. Then $v'(x) = 1$ for all x, and (22) suggests that $\lambda = 0$. Indeed, with risk neutrality, the first-best outcome can still be implemented by the informed bank. To see that, consider the following information rents of the bank:

$$R_0^{fb} = p[\theta_0 l_0^{fb} - c(l_0^{fb}, \theta_0)] + (1-p)[\theta_1 l_1^{fb} - c(l_1^{fb}, \theta_1)] - (1-p)\Phi(l_1^{fb}), \tag{28}$$

$$R_1^{fb} = p[\theta_0 l_0^{fb} - c(l_0^{fb}, \theta_0)] + (1-p)[\theta_1 l_1^{fb} - c(l_1^{fb}, \theta_1)] + p\Phi(l_1^{fb}). \tag{29}$$

It is easy to check that (ic_0), (ic_1), and (pc) are all satisfied by these information rents of the bank. As a result, the bank's incentive compatibility constraints do not conflict with the firm's participation constraint when the firm is not wealth constrained and the bank is risk neutral. Therefore, we can conclude that *ex ante* contracting never entails any allocative inefficiency when both parties are risk neutral under the case where the firm is not wealth constrained.[8]

On the contrary, it is easy to check by setting $v'(x) = 1$ for all x in section 4 that *ex ante* contracting still entails allocative inefficiency when both agents are risk neutral under the case where the firm is wealth constrained.[9]

6 Conclusions

This paper has examined optimal banking contracts using a simple example. It has pointed out that the firm's wealth constraint as well as risk preferences of both parties play an important role in financial contracting. If we assume that the firm is risk neutral and is wealth constrained and that the informed bank is strictly risk averse and makes the contractual offer at the *ex ante* stage, then the optimal contract entails the following:

1. Both the firm's *ex post* participation constraints in both states and the bank's incentive compatibility constraint in inefficient state θ_0 are binding.

2. No loan distortion for the lending that is obtained when inefficient state θ_0 realizes $l_0^* = l_0^{fb}$.

3. An upward distortion for the lending that is obtained when efficient state θ_1 realizes $l_1^* > l_1^{fb}$.

On the contrary, if we assume that the firm is not wealth constrained, then the optimal contract entails the following:

1. Both the firm's *ex ante* participation constraint and the bank's incentive compatibility constraint in state θ_1 are binding.

2. No loan distortion for the lending that is obtained when θ_1 realizes $l_1^* = l_1^{fb}$.

3. A downward distortion for the lending that is obtained when θ_0 realizes $l_0^* < l_0^{fb}$.

[8]This result is consistent with one of the case of risk neutrality of Laffont and Martimort (2002, chapter 9).

[9]This result is consistent with one of the case of infinite risk aversion of Laffont and Martimort (2002, chapter 9).

Finally, different conclusions are reached, if the bank as well as the firm is risk neutral. That is, no allocative inefficiency is obtained in the case of the wealth-unconstrained firm, while allocative inefficiency is still obtained in the case of the wealth-constrained firm.

References

[1] Aghion, P. and Bolton, P. (1992), "An Incomplete Contracts Approach to Financial Contracting," *Review of Economic Studies* 59: 473 – 494.

[2] Bolton, P. and Dewatripont, M. (2005), *Contract Theory*. Cambridge, MA: MIT Press.

[3] Bolton, P. and Scharfstein, D. S. (1990), "A Theory of Predation Based on Agency Problems in Financial Contracting," *American Economic Review* 80: 93 – 106.

[4] Crémer, J. and McLean, R. P. (1985), "Optimal Selling Strategies under Uncertainty for a Discriminating Monopolist When Demands are Interdependent," *Econometrica*. 53: 345 – 361.

[5] Crémer, J. and McLean, R. P. (1988), "Full Extraction of the Surplus in Bayesian and Dominant Strategy Auctions," *Econometrica*. 56: 1247 – 1257.

[6] Hart, O. and Moore, J. (1989), "Default and Renegotiation: A Dynamic Model of Debt," *Quarterly Journal of Economics* 113: 1 – 41.

[7] Hart, O. and Moore, J. (1994), "A Theory of Debt Based on the Inalienability of Human Capital," *Quarterly Journal of Economics* 109: 841 – 879.

[8] Hart, O. and Moore, J. (1998), "Default and Renegotiation: A Dynamic Model of Debt," *Quarterly Journal of Economics* 113: 1 – 41.

[9] Laffont, J.-J. and Martimort, D. (2002), *The Theory of Incentives: The Principal-Agent Model*. Princeton, NJ: Princeton University Press.

[10] McAfee, R. P. and Reny, P. J. (1992), "Correlated Information and Mechanism Design," *Econometrica*. 60: 395 – 421.

[11] Mirrlees, J. A. (1971), "An Exploration in the Theory of Optimum Income Taxation," *Review of Economic Studies* 38: 175 – 208.

[12] Neeman, Z. (2004), "The Relevance of Private Information in Mechanism Design," *Journal of Economic Theory*. 117: 55 – 77.

[13] Salanié, B. (2005), *The Economics of Contracts: A Primer*, 2nd ed.. Cambridge, MA: The MIT Press.

[14] Spence, A. M. (1973), "Job Market Signaling," *Quarterly Journal of Economics* 87: 355 – 374.

[15] Spence, A. M. (1974), *Market Signaling: Informational Transfer in Hiring and Related Screening Processes*. Cambridge, MA: Harvard University Press.